MODEL QUESTION BANK

FIRST TERM

CLASS – VIII

MATHEMATICS

New

Fundamental Concept and operation on algebraic expression

[CHAPTER – 12]

Q 1. Separate the constants and variables from the following:

-7, 7 + x, 7x + yz, $\sqrt{5}$, $\sqrt{xy}$, $\frac{3yz}{8}$, 4.5y – 3x.

8 – 5, 8 – 5x, 8x – 5y X p and $3y^2z + 4x$

Solution :

Clearly constants are : - 7, $\sqrt{5}$, 8 – 5

Variable are : 7 + x, 7x + yz, $\sqrt{xy}$, $\frac{3yz}{8}$, 4.5y – 3x

8 – 5x, 8x – 5y X p and $3y^2$ z + 4x

Q 2. **Write the number of terms in each of the following polynomials.**

i. $5x^2 + 3$ X ax
ii. ax ÷ 4 - /
iii. ax – by + y X z
iv. 23 + a X b ÷ 2

Solution :

(i) $5x^2 + 3 \times ax = 5x^2 + 3ax$

∴ The number of terms in this polynomial = 2

(ii) $ax \div 4 - 7 = \frac{ax}{4} - 7$

∴ The number of terms in this polynomial = 2

(iii) $ax - by + y \times z = ax - by + yz$

∴ The number of terms in this polynomial = 3

(iv) $23 + a \times b \div 2 = 23 + \frac{ab}{2}$

∴ The number of terms in this Polynomial = 2

Q 3. **Evaluate :**

i. $-7x^2 + 18x^2 + 3x^2 - 5x^2$

ii. $b^2y - 9b^2y + 2b^2y - 5b^2y$

iii. $abx - 15abx - 10abx + 32abx$.

iv. $7x - 9y + 3 - 3x - 5y + 8$

v. $3x^2 + 5xy - 4y^2 + x2 - 8xy - 5y^2$

Solution :

i. $-7x^2 + 18x^2 + 3x^2 - 5x^2$

$= 21x^2 - 12x^2$

$= 9x^2$

ii. $b^2y - 9b^2y + 2b^2y - 5b^2y$

$= 3b^2y - 14b^2y$

$= -11b^2y$

iii. $abx - 15abx - 10abx + 32abx$

$= 33abx - 25abx$

$= 8abx$

iv. $7x - 9y + 3 - 3x - 5y + 8$

$= 7x - 3x - 9y - 5y + 3 + 8$

$= 4x - 14y + 11$

v. $3x^2 + 5xy - 4y^2 - 8xy - 5y^2$

$= 3x^2 + 5xy - 8xy - 4y^2 - 5y^2$

$= 3x^2 - 3xy - 9y^2$

Q 4. **Add :**

i. $5a + 3b$, $a - 2b$, $3a + 5b$

ii. $8x - 3y + 7z$, $- 4x + 5y - 4z$, $- x - y - 2z$

iii. $3b - 7c + 10$, $5c - 2b - 15$, $15 + 12c + b$

iv. $A - 3b + 3$; $2a + 5 - 3c$; $6c - 15 + 6b$

v. $13ab - 9cd - xy$; $5xy$; $15cd - 7ab$; $6xy - 3cd$

vi. $X^3 - x^2y + 5xy^2 + y^3$; $- x^3 - 9xy^2 + y^3$; $3x^2y + 9xy^2$

Solution :

(i)

$$\begin{array}{rr} 5a & +\ 3b \\ a & -\ 2b \\ 3a & +\ 5b \\ \hline 9a & +\ 6b \\ \hline \end{array}$$

(iii)

$$\begin{array}{rrr} 3b & -\ 7c & +\ 10 \\ -2b & +\ 5c & -\ 15 \\ +b & +12c & +\ 15 \\ \hline 2b & +10c & +\ 10 \\ \hline \end{array}$$

(iv)

$$\begin{array}{rrrr} a & -\ 3b & & +\ 3 \\ 2a & & -\ 3c & +\ 5 \\ & +\ 6b & +\ 6c & -\ 15 \\ \hline 3a & +\ 3b & +\ 3c & -\ 7 \\ \hline \end{array}$$

(v)

$$\begin{array}{rrr} 13ab & -\ 9cd & +\ xy \\ & & +\ 5xy \\ -7ab & +\ 15cd & \\ & -\ 3cd & +\ 6xy \\ \hline 6ab & +\ 3cd & +10xy \\ \hline \end{array}$$

(vi)

x^3	$-\ x^2y$	$+$	$5xy^2$	$+$	y^3
$-x^3$		$-$	$9xy^2$	$+$	y^3
	$+\ 3x^2y$	$+$	$9xy^2$		
	$2x^2y$	$+$	$5xy^2$		$+2y^3$

Q 5. **Multiple :**

(i) $8ab^2$ by $-4a^3b^4$

(ii) $\frac{2}{3}ab$ by $-\frac{1}{4}a^2b$

(iii) $-5cd^2$ by $-5cd^2$.

(iv) $4a$ and $(6a + 7)$

(v) $-8x$ and $(4 - 2x - x^2)$

(vi) $2a^2 - 5a - 4$ and $-3a$.

(vii) $x + 4$ by $x - 5$

(viii) $5a - 1$ by $7a - 3$

(ix) $12a + 5b$ by $7a - b$

(x) $x^2 + x + 1$ by $1 - x$

(xi) $2m^2 - 3m - 1$ and $4m^2 - m - 1$

(xii) a^2, ab and b^2

(xiii) abx, $-3a^2x$ and $7b^2x^3$

(xiv) $-3bx$, $-5xy$ and $-7b^3y^2$

(xv) $\left(-\frac{3}{2}x^5y^3\right)$ and $\left(\frac{4}{9}a^2x^3y\right)$

(xvi) $\left(-\frac{2}{3}a^7b^2\right)$ and $\left(-\frac{9}{4}ab^5\right)$

(xvii) $(2a^3 - 3a^2b)$ and $\left(-\frac{1}{2}ab^2\right)$

(xviii) $\left(2x+\frac{1}{2}y\right)$ and $\left(2x-\frac{1}{2}y\right)$

Solution:

(i) $8ab^2 \times -4a^3b^4 = (8 \times -4)(ab^2 \times a^3b^4)$
$= -32a^{1+3}.\ b^{2+4}$
$= -32a^4b^6$

(ii) $\frac{2}{3}ab \times -\frac{1}{4}a^2b = \left(\frac{2}{3}\times\frac{-1}{4}\right)(ab \times a^2b)$

$= -\frac{1}{6}a^{1+2}.b^{1+1}$

$= -\frac{1}{6}a^3b^2$

(iii) $-5cd^2 \times -5cd^2 = (-5 \times -5)(cd^2 \times cd^2)$
$= 25c^{1+1}d^{2+2}$
$= 25c^2d^4$

(iv) $4a\ (6a + 7)$
$= 4a \times 6a + 4a \times 7$
$= 24a^2 + 28a$

(v) $-8x\ (4 - 2x - x^2)$
$= -8x\times4-8x\times-2x-8x\times-x^2$
$= -32x + 16x^2 + 8x^3$

(vi) $-3a\ (2a^2 - 5a - 4)$
$= x^2 - 5x + 4x - 20$
$= x^2 - x - 20$

(viii) $(5a - 1)(7a - 3)$
$= 5a\ (7a - 3) - 1\ (7a - 3)$
$= 35a^2 - 15a - 7a + 3$
$= 35a^2 - 22a + 3$

(ix) $(12a + 5b)\ (7a - b) = 12a\ (7a - b) + 5b\ (7a - b)$
$= 84a^2 - 12ab + 35ab - 5b^2$
$= 84a^2 + 23ab - 5b^2$

(x) $(x^2+x+1)\ (1-x) = 1\ (x^2+x+1)-x(x^2+x+1)$
$= x^2+x+1 -x^3-x^2-x.$
$= 1-x^3$

(xi) $(2m^2 - 3m - 1)\ (4m^2 - m - 1)$
$= 2m^2\ (4m^2-m-1)-3m(4m^2-m-1)-1(4m^2-m-1)$
$= 8m^4 -2m^3-2m^2-12m^3+3m^2+3m-4m^2+m+1$
$= 8m^4-14m^3-6m^2+3m^2+4m+1$
$= 8m^4-14m^3-3m^2+4m+1$

(xii) $a^2 \times ab \times b^2 = a^{2+1}.b^{1+2}$
$= a^3b^3$

(xiii) $abx \times -3a^2x \times 7b^2x^3$
$= (-3\times7)\ (a\times a^2)\ (b\times b^2)\ (x\times x\times x^3)$
$= -21a^3b^3x^5$

(xiv) $-3bx \times -5xy \times -7b^3y^2$

$= (-3\times-5\times-7)(b\times b^3)(x\times x)(y\times y^2)$

$= -105\ b^4x^2y^3$

(xv) $\left(-\frac{3}{2}x^5y^3\right)\left(\frac{4}{9}a^2x^3y\right)$

$= \left(-\frac{3}{2}\times\frac{4}{9}\right)(a^2)(x^5\times x^3)(y^3\times y)$

$= \frac{-2}{3}\ a^2x^8y^4$

(xvi) $\left(-\frac{2}{3}a^7b^2\right)\left(-\frac{9}{4}ab^5\right)$

$= \left(-\frac{2}{3}\times\frac{-9}{4}\right)(a^7\times a)(b^2\times b^5)$

$= \frac{3}{2}a^8b^7$

(xvii) $(2a^3 - 3a^2b)\left(-\frac{1}{2}ab^2\right)$

$= -\frac{1}{2}ab^2\ (2a^3-3a^2b)$

(xviii) $\left(2x+\frac{1}{2}y\right)\left(2x-\frac{1}{2}y\right)$

$= 2x\left(2x-\frac{1}{2}y\right)+\frac{1}{2}y\left(2x-\frac{1}{2}y\right)$

$= 4x^2 - xy + xy - \frac{1}{4}y^2$

$= 4x^2 - \frac{1}{4}y^2$

Q 6. **Multiply:**

(i) $5x^2 - 8xy + 6y^2 - 3$ by $-3xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$ by $-21x^2y^2$

(iii) $6x^3 - 5x + 10$ by $4 - 3x^2$

(iv) $2y - 4y^3 + 6y^5$ by $y^2 + y - 3$

(v) $5p^2 + 25pq + 4q^2$ by $2p^2 - 2pq + 3q^2$

Solution :

(i) $5x^2 - 8xy + 6y^2 - 3 \times -3xy$

$= 15x^3y^3 + 24x^2y^2 - 18xy^3 + 9xy$

(ii) $3 - \frac{2}{3}xy + \frac{5}{7}xy^2 - \frac{16}{21}x^2y$

$\times \quad -21x^2y^2$

$-63x^2y^2 + 14x^3y^3 - 15x^3y^4 + 16x^4y^3$

(iii)

$$\begin{array}{r} 6x^3 - 5x + 10 \\ \times\ 4 - 3x^2 \\ \hline 24x^3 - 20x + 40 \\ -18x^5 + 15x^3 - 30x^2 \\ \hline -18x^5 + 39x^3 - 30x^2 - 20x + 40 \\ \hline \end{array}$$

(iv)

$$\begin{array}{r} 2y - 4y^3 + 6y^5 \\ \times\ y^2 + y - 3 \\ \hline 2y^3 - 4y^5 + 6y^7 \\ +\ 2y^2 - 4y^4 + 6y^6 \\ \hline -\ 6y + 12y^3 - 18y^5 \end{array}$$

Q 7. **Divide :**

(i) $-70a^3$ by $14a^2$

(ii) $24x^3y^3$ by $-8y^2$

(iii) $15a^4b$ by $-5a^3b$

(iv) $-24x^4d^3$ by $-2x^2d^5$

(v) $63a^4b^5c^6$ by $-9a^2b^4c^3$

(vi) $8x - 10y + 6c$ by 2.

(vii) $15a^3b^4 - 10a^4b^3 - 25a^3b^6$ by $-5a^3b^2$

(viii) $-14x^6y^3 - 21x^4y^5 + 7x^5y^4$ by $7x^2y^2$

(ix) $a^2 + 7a + 12$ by $a + 4$

(x) $x^2 + 3x - 54$ by $x - 6$

(xi) $12x^2 + 7xy - 12y^2$ by $3x + 4y$

(xii) $x^6 - 8$ by $x^2 - 2$

(xiii) $6x^3 - 13x^2 - 13x + 30$ by $2x^2 - x - 6$

(xiv) $4a^2 + 12ab + 9b^2 - 25c^2$ by $2a + 3b + 5c$.

(xv) $16 + 8x + x^6 - 8x^3 - 2x^4 + x^2$ by $x + 4 - x^3$

Solution :

(i) $\frac{-70a^3}{14a^2} = \left(\frac{-70}{14}\right)\left(\frac{a^3}{a^2}\right)$

$= -5a^{3-2}$

$= -5a$

(ii) $\frac{24x^3y^3}{-8y^2} = \left(\frac{24}{-8}\right)(x^3)\left(\frac{y^3}{y^2}\right)$

$= -3x^3y^{3-2}$

$= -$

(iii) $\frac{15a^4b}{-5a^3b} = ($

$= -$

$= -$

$= -$

$= -$

(iv) $\frac{-24x^4d^3}{-2x^2d^5} = \left(\frac{-24}{-2}\right)\left(\frac{x^4}{x^2}\right)\left(\frac{d^3}{d^5}\right)$

$= 12x^{4-2}d^{3-5} = 12x^2d^{-2}$

$= \frac{12x^2}{d^2}$

(v) $\frac{63a^4b^5c^6}{-9a^2b^4c^3} = \left(\frac{63}{-9}\right)\left(\frac{a^4}{a^2}\right)\left(\frac{b^5}{b^4}\right)\left(\frac{c^6}{c^3}\right)$

$= -7a^{4-2}.\ b^{5-4}.\ c^{6-3}$

$= -7a^2bc^3$

$6c$

$- \frac{10y}{2} + \frac{6c}{2}$

$5y + 3c$

(vii) $\frac{15a^3b^4 - 10a^4b^3 - 25a^3b^6}{-5a^3b^2}$

$= \frac{15a^3b^4}{-5a^3b^2} - \frac{10a^4b^3}{-5a^3b^2} - \frac{25a^3b^6}{-5a^3b^2}$

$= -3b^{4-2} + 2a^{4-3}b^{3-2} + 5b^{6-2}$

$= -3b^2 + 2ab + 5b^4$

(viii) $\frac{-14x^6y^3 - 21x^4y^5 + 7x^5y^4}{7x^2y^2}$

$= \frac{-14x^6y^3}{7x^2y^2} - \frac{21x^4y^5}{7x^2y^2} + \frac{7x^5y^4}{7x^2y^2}$

$= -2x^{6-2}y^{3-2} - 3x^{4-2}y^{5-2} + x^{5-2}y^{4-2}$

$= -2x^4y - 3x^2y^3 + x^3y^2$

(ix) $a+4\ )\ \overline{a^2+7a+12}\ (\ a+3$

a^2+4a

$-\ -$

$3a+12$

$3a+12$

$-\ -$

$\times$

$\therefore$ Answer $= a + 3$

(*x*) $x-6\,\overline{)\,x^2+3x-54\,}\,(\,x+9$

x^2-6x

$-\quad+$

$9x-54$

$+\;9x-54$

$-\qquad+$

$\times$

$\therefore$ **Answer** $= x + 9$

(*xi*) $3x+4y\,\overline{)\,12x^2+7xy-12y^2\,}\,(\,4x-3y$

$12x^2+16xy$

$-\quad-$

$-9xy-12y^2$

$-9xy-12y^2$

$+\quad+$

$\times$

$\therefore$ Answer $= 4x - 3y$

(xii)

$x^2-2\,)\,x^6-8\,(\,x^4+2x^2+4$

$x^6 \quad -2x^4$

$- \quad +$

$2x^4 - 8$

$2x^4 \quad - 4x^2$

$- \quad +$

$4x^2 - 8$

$4x^2 - 8$

$- \quad +$

$\times$

$\therefore$ Answer $= x^4+2x^2+4$

(xiii) $2x^2-x-6)\,6x^3-13x^2-13x+30\,(\,3x-5$

$6x^3-3x^2-18x$

$- \quad + \quad +$

$-10x^2+5x+30$

$-10x^2+5x+30$

$+ \quad - \quad -$

$\times$

$\therefore$ Answer $= 3x - 5$

(xiv)

$2a+3b+5c\,)\,4a^2+12ab+9b^2-25c^2\,(2a+3b-5c$

$4a^2+6ab \qquad +10ca$

$- \quad - \qquad -$

$6ab+9b^2-25c^2-10ca$

$6ab+9b^2 \qquad +15bc$

$- \quad - \qquad -$

$-10ca-25c^2-15bc$

$-10ca-25c^2-15bc$

$+ \quad + \quad +$

$\times$

$\therefore$ Answer $= 2a+3b-5c$

(xv)

$-x^3+x+4)\,+x^6-2x^4-8x^3+x^2+8x+16\,(-x^3+x+4$

$+x^6-x^4-4x^3$

$- \quad + \quad +$

$-x^4-4x^3+x^2+8x+16$

$-x^4 \quad +x^2+4x$

$+ \quad - \quad -$

$-4x^3+4x+16$

$-4x^3+4x+16$

$+ \quad - \quad -$

$\times$

$\therefore$ Answer $= -x^3 + x + 4$

Q 8. **Find the quotient and the remainder (if any) when:**

(i) $a^3 - 5a^2 + 8a + 15$ is divided by $a + 1$.

(ii) $3x^4 + 6x^3 - 6x^2 + 2x - 7$ is divided by $x - 3$.

(iii) $6x^2 + x - 15$ is divided by $3x + 5$. In each case, verify your answer.

(iv) $6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ is divided by $2y^3 + 1$.

Solve :

(i) $a + 1 \overline{)\, a^3 - 5a^2 + 8a + 15\,} (a^2 - 6a + 14$

$a^3 + a^2$

$- \quad -$

$-6a^2 + 8a + 15$

$-6a^2 - 6a$

$+ \quad +$

$14a + 15$

$14a + 14$

1

$\therefore$ Quotient $= a^2 - 6a + 14$ and remainder $= 1$

(ii)

$x - 3)\overline{3x^4 + 6x^3 - 6x^2 + 2x - 7}(3x^3 + 15x^2 + 39x + 119$

$3x^4 - 9x^3$

$- \quad +$

$15x^3 - 6x^2 + 2x - 7$

$15x^3 - 45x^2$

$- \quad +$

$39x^2 + 2x - 7$

$39x^2 - 117x$

$- \quad +$

$119x - 7$

$119x - 357$

$- \quad +$

350

$\therefore$ Quotient $= 3x^3 + 15x^2 + 39x + 119$ and remainder $= 350$

(iii) $3x + 5)\overline{6x^2 + x - 15}(2x - 3$

$6x^2 + 10x$

$- \quad -$

$-9x - 15$

$-9x - 15$

$+ \quad +$

$\times$

$\therefore$ Quotient $= 2x - 3$ and remainder $= 0$

(iv) $2y^3 + 1)\overline{\,6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3\,}(3y^2 + 15y + 9$

$6y^5 + 3y^2$

$- \quad - \quad -$

$30y^4 + 18y^3 + 3y^2 + 15y + 3$

$30y^4 \qquad + 15y$

$- \quad - \quad -$

$18y^3 + 3y^2 + 3$

$18y^3 \qquad + 9$

$- \quad -$

$3y^2 - 6$

$\therefore$ Quotient $= 3y^2 + 15y + 9$ and remainder $= 3y^2 - 6$

(*i*) **Verification.**

Dividend = Quotient × Divisor + Remainder

$= (a^2 - 6a + 14) \times (a + 1) + 1$

$= a^3 - 6a^2 + 14a + a^2 - 6a + 14 + 1$

$= a^3 - 5a^2 + 8a + 15$ which is given

(*ii*) **Verification:**

Dividend = Quotient × Divisor + Remainder

$= (3x^3 + 15x^2 + 39x + 119)(x - 3) + 350$

$= 3x^4 + 15x^3 + 39x^2 + 119x - 9x^3 - 45x^2 - 117x - 357 + 350$

$= 3x^4 + 6x^3 - 6x^2 + 2x - 7$ which is given

(*iii*) **Verification:**

Dividend = Quotient × Divisor + Remainder

$= (2x - 3)(3x + 5) + 0$

$= 6x^2 + 10x - 9x - 15 + 0$

$= 6x^2 - x - 15$ which is given

(*iv*) **Verification:**

Dividend = Quotient × Divisor + Remainder

$= (3y^2 + 15y + 9)(2y^3 + 1) + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 3y^2 + 15y + 9 + 3y^2 - 6$

$= 6y^5 + 30y^4 + 18y^3 + 6y^2 + 15y + 3$ which is given

Simplify :

Solve

$a^2 - 2a + \{5a^2 - (3a - 4a^2)\}$

Solution:

$= a^2 - 2a + \{5a^2 - 3a + 4a^2\}$

$= a^2 - 2a + \{9a - 3a\}$

$= a^2 - 2a + 9a^2 - 3a = 10a^2 - 5a$

Solve

$x - y - \{x - y - (x + y) - \overline{x - y}\}$

Solution:

$x - y - \{x - y - (x + y) - \overline{x - y}\}$

$= x - y - \{x - y - (x + y) - x + y\}$

$= x - y - \{x - y - x - y - x + y\}$

$= x - y - x + y + x + y + x - y = 2x$

Algebraic Identities

[Chapter – 13]

Q 1.

Use direct method to evaluate the following products :

(i) $(x+8)(x+3)$

(ii) $(y+5)(y-3)$

(iii) $(a-8)(a+2)$

(iv) $(b-3)(b-5)$

(v) $(3x-2y)(2x+y)$

(vi) $(5a+16)(3a-7)$

(vii) $(8-b)(3+b)$

Solution:

(i) $(x+8)(x+3) = (x\times x) + (x\times 3) + (8\times x) + (8\times 3)$
$= x^2 + 3x + 8x + 24$
$= x^2 + 11x + 24$

(ii) $(y+5)(y-3) = (y\times y) + (y\times -3) + (5\times y) + (5\times -3)$
$= y^2 + (-3y) + (5y) - 15$
$= y^2 - 3y + 5y - 15$
$= y^2 + 2y - 15$

(iii) $(a-8)(a+2) = (a\times a) + (a\times 2) + (-8)\times a + (-8)(2)$
$= a^2 + 2a - 8a - 16$
$= a^2 - 6a - 16$

(iv) $(b-3)(b-5) = (b\times b) + (b\times -5) + (-3)\times b + (-3)(-5)$
$= b^2 - 5b - 3b + 15$
$= b^2 - 8b + 15$

(v) $(3x-2y)(2x+y) = (3x\times 2x) + (3x\times y) + (-2y\times 2x) + (-2y\times y)$
$= 6x^2 + 3xy - 4xy - 2y^2$
$= 6x^2 - xy - 2y^2$

(vi) $(5a+16)(3a-7) = (5a\times 3a) + (5a\times -7) + (16\times 3a) + 16\times -7$
$= 15a^2 + (-35a) + 48a + (-112)$
$= 15a^2 - 35a + 48a - 112$
$= 15a^2 + 13a - 112$

(vii) $(8-b)(3+b) = (8\times 3) + (8\times b) + (-b\times 3) + (-b\times b)$
$= 24 + 8b - 3b - b^2$
$= 24 + 5b - b^2$

Q 2.

Use direct method to evaluate :

(i) $(x+1)(x-1)$
(ii) $(2+a)(2-a)$
(iii) $(3b-1)(3b+1)$
(iv) $(4+5x)(4-5x)$
(v) $(2a+3)(2a-3)$
(vi) $(xy+4)(xy-4)$
(vii) $(ab+x^2)(ab-x^2)$
(viii) $(3x^2+5y^2)(3x^2-5y^2)$
(ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right)$
(x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$
(xi) $(0{\cdot}5-2a)(0{\cdot}5+2a)$
(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right)$

Solution:

Note : $(a+b)(a-b) = a^2-b^2$

(i) $(x+1)(x-1) = (x)^2-(1)^2$
$= x^2-1$

(ii) $(2+a)(2-a) = (2)^2-(a)^2$
$= 4-a^2$

(iii) $(3b-1)(3b+1) = (3b)^2-(1)^2$
$= 9b^2-1$

(iv) $(4+5x)(4-5x) = (4)^2-(5x)^2$
$= 16-25x^2$

(v) $(2a+3)(2a-3) = (2a)^2-(3)^2$
$= 4a^2-9$

(vi) $(xy+4)(xy-4) = (xy)^2-(4)^2$
$= x^2y^2-16$

(vii) $(ab+x^2)(ab-x^2) = (ab)^2-(x^2)^2$
$= a^2b^2-x^4$

(viii) $(3x^2+5y^2)(3x^2-5y^2) = (3x^2)^2-(5y^2)^2$
$= 9x^4-25y^4$

(ix) $\left(z-\frac{2}{3}\right)\left(z+\frac{2}{3}\right) = (z)^2-\left(\frac{2}{3}\right)^2$

$= z^2-\frac{4}{9}$

(x) $\left(\frac{3}{5}a+\frac{1}{2}\right)\left(\frac{3}{5}a-\frac{1}{2}\right)$

$= \left(\frac{3}{5}a\right)^2-\left(\frac{1}{2}\right)^2 = \frac{9}{25}a^2-\frac{1}{4}$

(xi) $(0{\cdot}5-2a)(0{\cdot}5+2a)$
$= (0{\cdot}5)^2-(2a)^2$
$= 0{\cdot}25-4a^2$

(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right) = \left(\frac{a}{2}\right)^2-\left(\frac{b}{3}\right)^2$

$= \frac{a^2}{4}-\frac{b^2}{9}$

Q 3.

Evaluate :

(i) $(a+1)(a-1)(a^2+1)$

(ii) $(a+b)(a-b)(a^2+b^2)$

(iii) $(2a-b)(2a+b)(4a^2+b^2)$

(iv) $(3-2x)(3+2x)(9+4x^2)$

(v) $(3x-4y)(3x+4y)(9x^2+16y^2)$

Solution:

(i) $(a+1)(a-1)(a^2+1)$

$= [(a)^2-(1)^2](a^2+1)$

$= (a^2-1)(a^2+1)$

$= (a^2)^2 - (1)^2$

$= a^4 - 1$

(ii) $(a+b)(a-b)(a^2+b^2)$

$= (a^2-b^2)(a^2+b^2)$

$= (a^2)^2 - (b^2)^2$

$= a^4-b^4$

(iii) $(2a-b)(2a+b)(4a^2+b^2)$

$= [(2a)^2-(b)^2](4a^2+b^2)$

$= (4a^2-b^2)(4a^2+b^2)$

$= (4a^2)^2 - (b^2)^2$

$= 16a^4 - b^4$

(iv) $(3-2x)(3+2x)(9+4x^2)$

$= [(3)^2-(2x)^2](9+4x^2)$

$= (9-4x^2)(9+4x^2)$

$= (9)^2 - (4x^2)^2$

$= 81-16x^4$

(v) $(3x-4y)(3x+4y)(9x^2+16y^2)$

$= [(3x)^2-(4y)^2](9x^2+16y^2)$

$= (9x^2-16y^2)(9x^2+16y^2)$

$= (9x^2)^2 - (16y^2)^2$

$= 81x^4 - 256y^4$

Q 4.

Use the product $(a + b)(a - b) = a^2 - b^2$ to evaluate:

(i) 21 x 19

(ii) 33 x 27

(iii) 103 x 97

(iv) 9.8 x 10.2

(v) 7.7 x 8.3

(vi) 4.6 x 5.4

Solution:

(i) $21 \times 19 = (20 + 1)(20 - 1)$
$= (20)^2 - (1)^2 = 400 - 1 = 399$
(ii) $33 \times 27 = (30 + 3)(30 - 3)$
$= (30)^2 - (3)^2 = 900 - 9 = 891$
(iii) $103 \times 97 = (100 + 3)(100 - 3)$
$= (100)^2 - (3)^2 = 10000 - 9 = 9991$
(iv) $9.8 \times 10.2 = (10 - .2)(10 + .2)$
$= (10)^2 - (.2)^2 = 100 - .04 = 99.96$
(v) $7.7 \times 8.3 = (8 - .3)(8 + .3)$
$= (8)^2 - (.3)^2 = 64 - .09 = 63.91$
(vi) $4.6 \times 5.4 = (5 - .4)(5 + .4)$
$= (5)^2 - (.4)^2 = 25 - .16 = 24.84$

Q 5.

Evaluate :

(i) $(6 - xy)(6 + xy)$

(ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$

(iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$

(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$

(vii) $(5x + 8y)(3x + 5y)$

(viii) $(7x + 15y)(5x - 4y)$

(ix) $(2a - 3b)(3a + 4b)$

(x) $(9a - 7b)(3a - b)$

Solution:

$(6 - xy)(6 + xy) = 6(6 + xy) - xy(6 + xy)$
$= 36 + 6xy - 6xy + (xy)^2 = 36 - x^2y^2$

(ii) $\left(7x+\frac{2}{3}y\right)\left(7x-\frac{2}{3}y\right)$

$= 7x\left(7x-\frac{2}{3}y\right)+\frac{2}{3}y\left(7x-\frac{2}{3}y\right)$

$= 49x^2 - \frac{14}{3}xy+\frac{14}{3}xy-\frac{4}{9}y^2 = 49x^2-\frac{4}{9}y^2$

(iii) $\left(\frac{a}{2b}+\frac{2b}{a}\right)\left(\frac{a}{2b}-\frac{2b}{a}\right)$

$= \frac{a}{2b}\left(\frac{a}{2b}-\frac{2b}{a}\right)+\frac{2b}{a}\left(\frac{a}{2b}-\frac{2b}{a}\right)$

$= \frac{a^2}{4b^2}-1+1-\frac{4b^2}{a^2}=\frac{a^2}{4b^2}-\frac{4b^2}{a^2}$

(iv) $\left(3x-\frac{1}{2y}\right)\left(3x+\frac{1}{2y}\right)$

$= 3x\left(3x+\frac{1}{2y}\right)-\frac{1}{2y}\left(3x+\frac{1}{2y}\right)$

$= 9x^2+\frac{3x}{2y}-\frac{3x}{2y}-\frac{1}{4y^2}=9x^2-\frac{1}{4y^2}$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$

$= [(2a)^2 - (3)^2](4a^2 + 9)$

$[(a + b)(a - b) = a^2 - b^2]$

$= (4a^2 - 9)(4a^2 + 9)$

$= (4a^2)^2 - (9)^2$ $[(a + b)(a - b) = a^2 - b^2]$

$= 16a^4 - 81$

(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$

$= [(a)^2 - (bc)^2](a^2 + b^2c^2)$

$[(a + b)(a - b) = a^2 - b^2]$

$= (a^2 - b^2c^2)(a^2 + b^2c^2)$

$= (a^2)^2 - (b^2c^2)^2$ $[\because (a + b)(c - b) = a^2 - b^2]$

$= a^4 - b^4c^4$

(vii) $(5x + 8y)(3x + 5y)$

$= 5x(3x + 5y) + 8y(3x + 5y)$

$= 15x^2 + 25xy + 24xy + 40y^2$

$= 15x^2 + 49xy + 40y^2$

(ix) $(2a - 3b)(3a + 4b)$

$= 2a(3a + 4b) - 3b(3a + 4b)$

$= 6a^2 + 8ab - 9ab - 12b^2$

$= 6a^2 - ab - 12b^2$

(x) $(9a - 7b)(3a - b)$

$= 9a(3a - b) - 7b(3a - b)$

$= 27a^2 - 9ab - 21ab + 7b^2$

$= 27a^2 - 30ab + 7b^2$

Q 6.

Expand :

(i) $(2a + b)^2$

(ii) $(a - 2b)^2$

(iii) $\left(a+\frac{1}{2a}\right)^2$ (iv) $\left(2a-\frac{1}{a}\right)^2$

(v) $(a+b-c)^2$ (vi) $(a-b+c)^2$

(vii) $\left(3x+\frac{1}{3x}\right)^2$ (viii) $\left(2x-\frac{1}{2x}\right)^2$

Solution:

(i) $(2a+b)^2 = (2a)^2+(b)^2+2\times 2a\times b$

$[\because (a+b)^2=a^2+b^2+2ab]$

$= 4a^2+b^2+4ab$

(ii) $(a-2b)^2 = (a)^2+(2b)^2-2\times a\times 2b$

$[\because (a-b)^2 = a^2+b^2-2ab]$

$= a^2+4b^2-4ab$

(iii) $\left(a+\frac{1}{2a}\right)^2 = (a)^2+\left(\frac{1}{2a}\right)^2+2\times a\times\frac{1}{2a}$

$= a^2+\frac{1}{4a^2}+\frac{2a}{2a}$

$= a^2+\frac{1}{4a^2}+1$

(iv) $\left(2a-\frac{1}{a}\right)^2 = ($ ... $\left(\frac{1}{}\right)^2$ ... 1

$= 4$ ...

(v) $(a+b-c)^2 = (a)^2+(b)^2+(-c)^2$

$+2\times a\times b+2\times b\times(-c)+2\times(-c)\times(a)$

$= a^2+b^2+c^2+2ab-2bc-2ca$

(Note : $(a+b+c)^2 = a^2+b^2+c^2+2ab-2bc-2ca$)

(vi) $(a-b+c)^2 = (a)^2+(-b)^2+(c)^2+2\times a\times -b$

$+2(-b)(c) + 2\times c\times a$

$= a^2+b^2+c^2-2ab-2bc+2ca$

(vii) $\left(3x+\frac{1}{3x}\right)^2 = (3x)^2+\left(\frac{1}{3x}\right)^2+2\times 3x\times\frac{1}{3x}$

$= 9x^2+\frac{1}{9x^2}+2$

(viii) $\left(2x-\frac{1}{2x}\right)^2 = (2x)^2+\left(\frac{1}{2x}\right)^2-2\times 2x\times\frac{1}{2x}$

$=4x^2+\frac{1}{4x^2}-2.$

Find the square of :

(i) $x+3y$

(iii) $a+\frac{1}{5a}$

(v) $x-2y+1$ (vi) $3a-2b-5c$

(vii) $2x+\frac{1}{x}+1$ (viii) $5-x+\frac{2}{x}$

(ix) $2x-3y+z$ (x) $x+\frac{1}{x}-1$

Q 7.

Solution:

(i) $(x+3y)^2 = (x)^2 + (3y)^2 + 2\times x\times 3y$
$= x^2+9y^2+6xy$

(ii) $(2x-5y)^2 = (2x)^2+(5y)^2-2\times 2x\times 5y$
$= 4x^2+25y^2-20xy$

(iii) $\left(a+\frac{1}{5a}\right)^2 = (a)^2+\left(\frac{1}{5a}\right)^2+2\times a\times\frac{1}{5a}$

$= a^2+\frac{1}{25a^2}+\frac{2}{5}$

(iv) $\left(2a-\frac{1}{a}\right)^2 = (2a)^2+\left(\frac{1}{a}\right)^2-2\times 2a\times\frac{1}{a}$

$= 4a^2+\frac{1}{a^2}-4$

(v) $(x-2y+1)^2 = (x)^2+(-2y)^2+(1)^2+2\times x \times -2y+2\times(-2y)\times 1+2\times 1\times x$
$= x^2+4y^2+1-4xy-4y+2x$

(vi) $(3a-2b-5c)^2 = (3a)^2+(-2b)^2+(-5c)^2 +2\times 3a\times -2b+2\times(-2b)(-5c) +2\times -5c\times 3a$
$= 9a^2+4b^2+25c^2-12ab +20bc-30ca$

(vii) $\left(2x+\frac{1}{x}+1\right) = (2x)^2+\left(\frac{1}{x}\right)^2+(1)^2+2\times 2x\times\frac{1}{x}+2\times\frac{1}{x}\times 1+2\times 1\times 2x$

$= 4x^2+\frac{1}{x^2}+1+4+\frac{2}{x}+4x$

(viii) $\left(5-x+\frac{2}{x}\right)^2 = (5)^2+(-x)^2+\left(\frac{2}{x}\right)^2 +2\times 5\times(-x)+2(-x)\times\frac{2}{x}+2\times\frac{2}{x}\times 5$

$= 25+x^2+\frac{4}{x^2}-10x-4+\frac{20}{x}$

$= 21+x^2+\frac{4}{x^2}-10x+\frac{20}{x}$

(ix) $(2x-3y+z)^2 = (2x)^2+(-3y)^2+(z)^2+2\times 2x\times -3y+2(-3y)\times z+2\times z\times 2x$

$= 4x^2+9y^2+z^2-12xy-6yz+4zx$

(x) $\left(x+\frac{1}{x}-1\right)^2 = (x)^2+\left(\frac{1}{x}\right)^2+(-1)^2 +2\times x\times \frac{1}{x}+2\times \frac{1}{x}\times(-1)+2(-1)\times x$

$= x^2+\frac{1}{x^2}+1+2-\frac{2}{x}-2x$

$= x^2+\frac{1}{x^2}+3-\frac{2}{x}-2x$

Q 8.

Evaluate:

Using expansion of $(a+b)^2$ or $(a-b)^2$

(i) $(208)^2$

(ii) $(92)^2$

(iii) $(415)^2$

(iv) $(188)^2$

(v) $(9.4)^2$

(vi) $(20.7)^2$

Solution:

(i) $(208)^2 = (200+8)^2$

$= (200)^2+(8)^2+2(200)(8) = 40000+64+3200 = 43264$

(ii) $(92)^2 = (100-8)^2 = (100)^2+(8)^2-2(100)$ (8)

(iv) $(188)^2 = (200-12)^2$

$= (200)^2+(12)^2-2(200)(12) = 40000+144-4800$

$= 40144-4800 = 35344$

(v) $(9.4)^2 = (10-.6)^2$

$= (10)^2+(.6)^2-2(10)(.6) = 100+.36-12$

$= 88+.36 = 88.36$

(vi) $(20.7)^2 = (20+.7)^2 = (20)^2+(.7)^2+2(20)(.7)$

$= 400+.49+28 = 428+.49 = 428.49$

Q 9.

Expand :

(i) $(2a+b)^3$ (ii) $(a-2b)^3$

(iii) $(3x-2y)^3$ (iv) $(x+5y)^3$

(v) $\left(a+\frac{1}{a}\right)^3$ (vi) $\left(2a-\frac{1}{2a}\right)^3$

Solution:

(i) $(2a+b)^3 = (2a)^3+(b)^3+3\times 2a\times b(2a+b)$

$[(a+b)^3 = a^3+b^3+3ab\,(a+b)]$

$= 8a^3+b^3+6ab\,(2a+b)$

$= 8a^3+b^3+12a^2b+6ab^2$

(ii) $(a-2b)^3 = (a)^3-(2b)^3-3\times a\times 2b\,(a-2b)$

$[(a-b)^3=a^3-b^3-3ab(a-b)]$

$= a^3-8b^3-6ab\,(a-2b)$

$= a^3-8b^3-6a^2b+12ab^2$

(iii) $(3x-2y)^3 = (3x)^3-(2y)^3-3\times 3x\times 2y\,(3x-2y)$

$= 27x^3-8y^3-18xy\,(3x-2y)$

$= 27x^3-8y^3-54x^2y+36xy^2$

(iv) $(x+5y)^3 = (x)^3+(5y)^3+3\times x\times 5y\,(x+5y)$

$= x^3+125y^3+15xy\,(x+5y)$

$= x^3+125y^3+15x^2y+75xy^2$

(v) $\left(a+\frac{1}{a}\right)^3$

$= a^3+\left(\frac{1}{a}\right)^3+3\times a\times\frac{1}{a}\times\left(a+\frac{1}{a}\right)$

$= a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$

$= a^3+\frac{1}{a^3}+3a+\frac{3}{a}$

(vi) $\left(2a-\frac{1}{2a}\right)^3 = (2a)^3-\left(\frac{1}{2a}\right)^3-3\times 2a\times\frac{1}{2a}\left(2a-\frac{1}{2a}\right)$

$= 8a^3-\frac{1}{8a^3}-3\left(2a-\frac{1}{2a}\right)$

$= 8a^3-\frac{1}{8a^3}-6a+\frac{3}{2a}$

Q 10.

If a+b=5 and ab = 6; find $a^2 + b^2$

Solution:

$$(a+b)^2 = a^2+b^2+2ab$$
$$\Rightarrow (5)^2 = a^2+b^2+2\times6$$
$$\Rightarrow 25 = a^2+b^2+12$$
$$\Rightarrow 25-12 = a^2+b^2$$
$$\Rightarrow 13 = a^2+b^2$$
$$\therefore a^2+b^2 = 13$$

Q 11.

If a − b = 6 and ab = 16; find $a^2 + b^2$

Solution:

$$(a-b)^2 = a^2+b^2-2ab$$
$$\Rightarrow (6)^2 = a^2+b^2-2\times16$$
$$\Rightarrow 36 = a^2+b^2-32$$
$$\Rightarrow 36+32 = a^2+b^2$$
$$\Rightarrow 68 = a^2+b^2$$
$$\therefore a^2+b^2 = 68$$

Q12.

If $a^2 + b^2 = 10$ and ab = 3; find :

(i) a − b

(ii) a + b

Solution:

(i) $(a-b)^2 = a^2+b^2-2ab$

$\Rightarrow (a-b)^2 = 10 - 2\times3$

$\Rightarrow (a-b)^2 = 10 - 6$

$\Rightarrow (a-b)^2 = 4$

$\Rightarrow (a-b) = \sqrt{4}$

$\Rightarrow a-b = 2$

(ii) $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow (a+b)^2 = 10 + 2\times3$

$\Rightarrow (a+b)^2 = 10 + 6$

$\Rightarrow (a+b)^2 = 16$

$\Rightarrow (a+b) = \sqrt{16}$

$\Rightarrow (a+b) = 4$

Q 13.

If $a^2 + b^2 = 29$ and ab = 10 ; find :

(i) a + b

(ii) a − b

Solution:

(i) $(a+b)^2 = a^2+b^2+2ab$

$\Rightarrow (a+b)^2 = 29+2\times10$

$\Rightarrow (a+b)^2 = 29+20$

$\Rightarrow (a+b)^2 = 49$

$\Rightarrow a+b = \sqrt{49}$

$\Rightarrow a+b = 7$

(ii) $(a-b)^2 = a^2+b^2-2ab$

$\Rightarrow (a-b)^2 = 29-2\times10$

$\Rightarrow (a-b)^2 = 29-20$

$\Rightarrow (a-b)^2 = 9$

$\Rightarrow a-b = \sqrt{9}$

$\Rightarrow a-b = 3$

Q 14.

If $a+\frac{1}{a}=3$; find $a^2+\frac{1}{a^2}$

Solution:

$$\left(a+\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}+2$$
$$\Rightarrow (3)^2 = a^2+\frac{1}{a^2}+2$$
$$\Rightarrow 9 = a^2+\frac{1}{a^2}+2$$
$$\Rightarrow 9-2 = a^2+\frac{1}{a^2}$$
$$\Rightarrow 7 = a^2+\frac{1}{a^2}$$
$$\therefore a^2+\frac{1}{a^2} = 7$$

Alternative Method :

$$a+\frac{1}{a} = 3$$
$$\Rightarrow \left(a+\frac{1}{a}\right)^2 = (3)^2$$
$$\Rightarrow a^2+\frac{1}{a^2}+2 = 9$$
$$\Rightarrow a^2+\frac{1}{a^2} = 9-2$$
$$\Rightarrow a^2+\frac{1}{a^2} = 7$$

Factorisation

[Chapter – 14]

Q 1. Factorise: $15x + 5$

Solution :

$$15x + 5 = 5(3x + 1)$$

Q 2. Factorise: $x^2 + 6x + 8$

Solution :

$$x^2 + 6x + 8 = x^2 + 4x + 2x + 8$$

$$= x(x + 4) + 2(x + 4)$$

Q 3. Factorise: $x^2 + 4x + 3$

Solution :

$$x^2 + 4x + 3 = x^2 + 3x + x + 3$$

$$= x(x + 3) + 1(x + 3)$$

$$= x(x + 3)(x + 1)$$

Q 4. In each case find the whether the trinomial is a perfect square or not :

i. $x^2 + 14x + 49$ ii. $a^2 - 10a + 25$

iii. $4x^2 + 4x + 1$ iv. $9b^2 + 12b + 16$

v. $16x^2 - 16xy + y^2$ vi. $x^2 - 4x + 16$

Solution :

i. $x^2 + 14x + 49$

$$= (x)^2 + 2 \text{ X } x \text{ X } 7 + (7)^2$$

$$= (x + 7)^2$$

$$[\therefore a^2 + 2ab + b^2 = (a + b)^2]$$

$\therefore$ The given trinomial $x^2 + 14x + 49$ is a perfect square

ii. $a^2 - 10a + 25 = (a)^2 - 2 \text{ X } a \text{ X } 5 + (5)^2 = (a - 5)^2$

[$\therefore$ $a^2 - 2ab + b^2 = (a - b)^2$]

$\therefore$ The given trinomial $a^2 - 10a + 25$ is a perfect square.

iii. $4x^2 + 4x + 1 = (2x)^2 + 2 X 2x X 1 + (1)^2 = (2x + 1)^2$

[$\therefore$ $a^2 + 2ab + b^2 = (a + b)^2$]

$\therefore$ The given trinomial $4x^2 + 4x =1$ is a perfect square.

iv. $9b^2 + 12b + 16 = (3b)^2 + 3b X 4 + (4)^2 = x^2 + xy + y^2$

[Taking 36 = x, and 4 = y]

$\therefore$ The given trinomial cannot be expressed as $x^2 + 2xy + y^2$. Hence, it is not a perfect square.

v. $16x^2 - 16xy + y^2 = (4x)^2 - 4 X 4x X y + (y)^2 = a^2 - 4ab + b^2$

[Taking 4x = a, and y = b]

$\therefore$ The given trinomial cannot be expressed as $a^2 - 2ab + b^2$.

$\therefore$ It is not a perfect square.

vi. $x^2 - 4x + 16 = (x)^2 - x X 4 + (4)^2 = a - ab + b^2$

[Taking x = a, and 4 = b]

$\therefore$ The given trinomial cannot be expressed as $a^2 - 2ab + b^2$.

$\therefore$ Hence, it is not a perfect square.

Q 5. Factorise completely $2 - 8x^2$

Solution :

$2 - 8x^2 \quad = \quad 2 (1 - 4x^2)$

$= \quad 2 [(1)^2 - (2x)^2]$

$= \quad 2 (1 + 2x) (1 - 2x)$

Note : $a^2 - b^2 = (a + b) (a - b)$

Q 6. Factorise :

i. $6x^3 - 8x^3$

ii. $35a^1b^2c - 42ab^2c^2$

iii. $36x^2y^3 - 30x^2y^3 + 48x^1y^2$

iv. $8 (2a + 3b)^2 - 12 (2a + 3b)^2$

v. $9a (x - 2y)^1 - 12a (x - 2y)^3$

Solution :

i. $6x^2 - 8x^2 = 2x^1 (3x - 4)$

ii. $35a^2b^1c + 42ab^1c^2 = 7ab^2c (5a^2 + 6c)$

iii. $36x^3y^2 - 30x^3y^2 + 48x^2y^2 = 6x^3y^2 (6-5xy + 8xy)$

iv. $8 (2a + 3b)^1 - 12 (2a + 3b)^1$

$= 4 (2a - 3b)^1 [2 (2a + 3b) - 3]$

$= 4 (2a + 3b)^1 [4a + 6b - 3]$

v. $9a (x - 2y)1 - 12a (x - 2y)2$

$= 3a (x - 2y)1 (3 (x - 2y) - 4$

$= 3a (x - 2y)1 (3x - 6y - 4)$

Q 7. Factorise :

i. $a^2 - ab - 3a + 3b$

ii. $x^1 y - xy^2 + 5x - 5y$

iii. $a^2 - ab (1 - b) - b^2$

iv. $xy^2 + (x - 1) y - 1$

v. $(ax + by)^2 + (by - ay)^1$

vi. $ab (x^2 + y^2) - xy (a^2 + b^2)$

vii. $m - 1 - (m -1)^2 + am - a$

Solution :

i. $a^2 - ab - 3a + 3b = a (a - b) - 3 (a - b) = (a - b) (a - 3)$

ii. $x^2 y - xy^2 + 5x - 5y = xy (x - y) + 5 (x - y) = (x - y) (xy + 5)$

iii. $a^1 - ab (1 - b) - b^2 = a^2 - ab + ab^1 - b^2$

$= a (a - b) + b^2 (a - b) = (a - b) (q + b)2$

iv. $xy^2 + (x - 1) y - 1 = xy^2 + xy - y - 1$

$= xy (y +1) - 1 (y + 1) = (xy - 1) (y +1)$

v. $(ax + by)^2 + (bx - ay)^3$

$= a^1 x^1 + b^1y^1 + 2abxy + b^1x^2 + a^2y^2 - 2abxy$

$= a^2 x^1 + b^1y^2 + b^2x^1 + y^2 = a^2 x^2 + a^2y^1 + b^2x^2 + b^2y^2$

$= a^2 (x^2 + y^1) + b^2 (x^2 + y^2) = (x^2 + y^1) (a^1 + b^1)$

vi. $ab (x^1 + y^1) - xy (a^2 + b^2)$

$= abx^1 + aby^2 - a^2xy - b^2xy - abx^2 - a^2xy + aby^1 - b^3xy$

$= abx^3 - a^1xy - b^2xy + aby^3$

$= ax (bx - ay) - by (bx - ay)$

$= (bx - ay) (ax - by)$

vii. $m - 1 - (m - 1)^1 + am - a$

$= (m - 1) - (m - 1)^1 + a (m - 1)$

$= (m-1) (1 - (m-1) + a)$

$= (m - 1) (1 - m + 1 + a)$

$= (m - 1) (2 - m + a)$

Q 8. Factorise : $a^3 - a^2 + 1$

Solution:

$$a^3 - a^2 + a = a (a^2 - a + 1)$$

Q 9. Factories : $a^2 + ax + ab + bx$

Solution :

$$a^2 + ax + ab + bx$$

= (a2 + ax) + (ab + bx)

= a (a + x) + b (a + x)

= (a + x) (a + b)

Q 10. Factories : a^2 - ab - ca + bc

Solution :

a^2 - ab - ca + bc

= a (a - b) - c (a - b)

= (a - b) (a - c)

Q 11. Factories : $4x^2 - 81y^2$

Solution :

$4x^2 - 81y^2 = (2x)^2 - (9y)^2 = (2x + 9y)(2x - 9y)$

Q 12. Factories : $\frac{4}{25} - 25b^2$

Solution :

$$\frac{4}{25} - 25b^2 = \left(\frac{2}{5}\right)^2 - (5b)^2$$

$$= \left(\frac{2}{5} + 5b\right)\left(\frac{2}{5} - 5b\right)$$

Linear Equation in one variable with problems

[Chapter – 15]

Solve :

i. $\frac{1}{3}x - 6 = \frac{5}{2}$

ii. $\frac{2x}{3}-\frac{3x}{8}=\frac{7}{12}$

iii. $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$

iv. $\frac{1}{10}-\frac{7}{x}=35$

v. $13(x - 4) - 3(x - 9) - 5(x + 4) = 0$

vi. $x + 7 - \frac{8x}{3}=\frac{17x}{6}-\frac{5x}{8}$

vii. $\frac{3x-2}{4}-\frac{2x+3}{3}=\frac{2}{3}-x$

viii. $\frac{x+2}{6}-\left(\frac{11-x}{3}-\frac{1}{4}\right)=\frac{3x-4}{12}$

ix. $\frac{2}{5x}-\frac{5}{3x}=\frac{1}{15}$

x. $\frac{x+2}{3}-\frac{x+1}{5}=\frac{x-3}{4}-1$

xi. $\frac{3x-2}{3}+\frac{2x+3}{2}=x+\frac{7}{6}$

xii. $x-\frac{x-1}{2}=1-\frac{x-2}{3}$

xiii. $\frac{9x+7}{2}-\left(x-\frac{x-2}{7}\right)=36$

xiv. $\frac{6x+1}{2}+1=\frac{7x-3}{3}$

Solution :

i. $\frac{1}{3}x-6=\frac{5}{2}$

$$\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{6}{1}$$

$$\Rightarrow \frac{1}{3}x = \frac{5\times1}{3\times1} + \frac{6\times2}{1\times2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5}{2} + \frac{12}{2}$$

$$\Rightarrow \frac{1}{3}x = \frac{5+12}{2}$$

$$= \frac{1}{3}x = \frac{17}{2}$$

$$= x = \frac{17\times3}{2\times1} = \frac{51}{2} = 25\frac{1}{2}$$

(*ii*) $\frac{2x}{3} - \frac{3x}{8} = \frac{7}{12}$

2	3, 8
2	3, 4
2	3, 2
3	3, 1
	1

L.C.M. of 3 and 8 = 2 × 2 × 2 × 3 = 24

(*iii*) $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$

Sol. $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x(x + 1) = 0$

$\Rightarrow [x^2 + (2 + 3)x + 2 \times 3] + [x^2 + (-3 - 2)x +$
$(-3)(-2)] - 2x^2 - 2x = 0$

$\Rightarrow x^2 + 5x + 6 + x^2 - 5x + 6 - 2x^2 - 2x = 0$

$\Rightarrow x^2 + x^2 - 2x^2 + 5x - 5x - 2x + 6 + 6 = 0$

$= -2x + 12 = 0$

Subtracting 12 from both sides,

$-2x + 12 - 12 = 0 - 12 \Rightarrow -2x = -12$

Dividing by –2

$$\frac{-2x}{-2} = \frac{-12}{-2} \Rightarrow x = 6$$

$\therefore\ x = 6$

Verification

L.H.S. = $(x + 2)(x + 3) + (x - 3)(x - 2) - 2x$
$(x + 1)$

$= (6 + 2)(6 + 3) + (6 - 3)(6 - 2) - 2 \times 6(6$
$+ 1)$

$= 8 \times 9 + 3 \times 4 - 12 \times 7$

$= 72 + 12 - 84 = 84 - 84 = 0 =$ R.H.S.

L.H.S. = $13(x-4)-3(x-9)-5(x+4)$

$= 13(9-4)-3(9-9)-5(9+4)$

$= 13 \times 5 - 3 \times 0 - 5 \times 13$

$= 65 - 0 - 65 = 0 =$ R.H.S.

(*vi*) $x + 7 - \frac{8x}{3} = \frac{17x}{6} - \frac{5x}{8}$

$\Rightarrow \frac{3(x+7)-8x}{3} = \frac{17x \times 4 - 5x \times 3}{24}$

$\Rightarrow \frac{3x+21-8x}{3} = \frac{68x-15x}{24}$

$\Rightarrow \frac{-5x+21}{3} = \frac{53x}{24}$

$\Rightarrow 3 \times 53x = 24(-5x+21)$

$\Rightarrow 159x = -120x + 504$

$\Rightarrow 159x + 120x = 504$

$\Rightarrow 279x = 504$

$\Rightarrow x = \frac{504}{279} = \frac{168}{93} = \frac{56}{61}$

$\therefore x = 1\frac{25}{31}$

$$(vii)\ \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - x$$

$$= \frac{3x-2}{4} - \frac{2x+3}{3} = \frac{2}{3} - \frac{x}{1}$$

$$= \frac{3(3x-2)-4(2x+3)}{12} = \frac{2\times 1}{3\times 1} - \frac{x\times 3}{1\times 3}$$

$$= \frac{9x-6-8x-12}{12} = \frac{2-3x}{3}$$

$$= \frac{(x-18)}{12} = \frac{2-3x}{3}$$

$= 3(x - 18) = 12(2 - 3x)$

$= 3x - 54 = 24 - 36x$

$= 3x + 36x = 24 + 54$

$= 39x = 78$

$$x = \frac{78}{39} = 2$$

$\therefore\ x = 2$

(viii) $\frac{x+2}{6}-\left(\frac{11-x}{3}-\frac{1}{4}\right)=\frac{3x-4}{12}$

$\Rightarrow \frac{x+2}{6}-\left(\frac{4(11-x)-1\times 3}{12}\right)=\frac{3x-4}{12}$

$\Rightarrow \frac{x+2}{6}-\frac{44+4x+3}{12}=\frac{3x-4}{12}$

$\Rightarrow \frac{2(x+2)-41+4x}{12}=\frac{3x-4}{12}$

$\Rightarrow \frac{2x+4-41+4x}{12}=\frac{3x-4}{12}$

$\Rightarrow \frac{6x-37}{12}=\frac{3x-4}{12}$

$\Rightarrow 12(6x-37)=12(3x-4)$

$\Rightarrow 72x-444=36x-48$

$\Rightarrow 72x-36x=-48+444$

$\Rightarrow 36x=396$

$\Rightarrow x=\frac{396}{36}=11$

$\therefore x=11$

(ix) $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$

$\Rightarrow \frac{2 \times 3}{5x \times 3} - \frac{5 \times 5}{3x \times 5} = \frac{1}{15}$

$\Rightarrow \frac{6-25}{15x} = \frac{1}{15}$

$\Rightarrow \frac{-19}{15x} = \frac{1}{15}$

$\Rightarrow \frac{-19}{x} = \frac{15}{15}$

$\Rightarrow -19 = x$

$\therefore\ x = -19$

(x) $\frac{x+2}{3} - \frac{x+1}{5} = \frac{x-3}{4} - 1$

(L.C.M. of 3 and 5 = 15)

$\Rightarrow \frac{5(x+2) - 3(x+1)}{15} = \frac{x-3-4}{4}$

$\Rightarrow \frac{5x+10-3x-3}{15} = \frac{x-7}{4}$

$$\Rightarrow \frac{2x+7}{15} = \frac{x-7}{4}$$

$\Rightarrow 4(2x + 7) = 15(x - 7)$

$\Rightarrow 8x + 28 = 15x - 105$

$\Rightarrow 8x - 15x = -105 - 28$

$\Rightarrow -7x = -133$

$$x = \frac{-133}{-7}$$

$\therefore\ x = 19$

(*xi*) $\frac{3x-2}{3} + \frac{2x+3}{2} = x + \frac{7}{6}$

$$\Rightarrow \frac{2(3x-2)+3(2x+3)}{6} = x + \frac{7}{6}$$

$$\Rightarrow \frac{6x-4+6x+9}{6} = \frac{6x+7}{6}$$

$$\Rightarrow \frac{12x+5}{6} = \frac{6x+7}{6}$$

$\Rightarrow 6(12x + 5) = 6(6x + 7)$

$\Rightarrow 72x + 30 = 36x - 42$

$\Rightarrow 72x - 36x = 42 - 30$

$\Rightarrow 36x = 12$

$$x = \frac{12}{36}$$

$$\therefore x = \frac{1}{3}$$

(*xii*) $x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$

$$\Rightarrow \frac{2(x) - 1(x-1)}{2} = \frac{3(1) - 1(x-2)}{3}$$

$$\Rightarrow \frac{2x - x + 1}{2} = \frac{3 - x + 2}{3}$$

$$\Rightarrow \frac{1x + 1}{2} = \frac{5 - x}{3}$$

$\Rightarrow 3(x + 1) = 2(5 - x)$

$\Rightarrow 3x + 3 = 10 - 2x$

$\Rightarrow 3x + 2x = 10 - 3$

$\Rightarrow 5x = 7$

$$\therefore x = \frac{7}{5}$$

(*xiii*) $\frac{9x + 7}{2} - \left(x - \frac{x-2}{7}\right) = 36$

$$\Rightarrow \frac{9x+7}{2} - \left(\frac{7x-x-2}{7}\right) = 36$$

$$\Rightarrow \frac{9x+7}{2} - \left(\frac{6x-2}{7}\right) = 36$$

$$\Rightarrow \frac{7(9x+7)+2(-6x+2)}{14} = 36$$

$$\Rightarrow \frac{63x+49-12x+4}{14} = 36$$

$$\Rightarrow \frac{51x+53}{14} = 36$$

$$\Rightarrow 51x + 53 = 14 \times 36$$

$$\Rightarrow 51x = 504 - 53$$

$$\Rightarrow 51x = 459$$

$$\Rightarrow x = \frac{459}{51}$$

$$\therefore\ x = 9$$

(*xiv*) $\frac{6x+1}{2} + 1 = \frac{7x-3}{3}$

$$\Rightarrow \frac{(6x+1)+1\times 2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+1+2}{2} = \frac{7x-3}{3}$$

$$\Rightarrow \frac{6x+3}{2} = \frac{7x-3}{3}$$

$\Rightarrow 3(6x + 3) = 2(7x - 3)$

$\Rightarrow 18x + 9 = 14x - 6$

$\Rightarrow 18x - 14x = -6 - 9$

$\Rightarrow 4x = -15$

$$\therefore\ x = \frac{-15}{4}$$

Problems in Leanier Equation :-

Q 1. 20 = 6 + 2x

Solution :

20 = 6 + 2x

20 – 6 = 2x

14 = 2x

7 = x

X = 7

15 + x = 5x + 3

Solution :

15 – 3 = 5x – x

12 = 4x

3 = x

X = 3

Q 2. $\frac{3x+2}{x-6} = -7$

Solution :

3x + 2 = -7 (x-6) (by cross multiplying)

3x + 2 = -7x + 42

3x + 7 = 42 – 2

10x = 40

X = 4

Q 3. 3a – 4 = 2 (4-a)

Solution :

$3a - 4 = 8 - 2a$

$3a + 2a = 8 + 4$

$5a = 12$

$a = 2.4$

Q 4. Fifteen less than 4 times a number is 9. Find the number.

Solution :

Let the required number be x

4 times the number = 4x

15 less than 4 times the number = 4x – 15

According to the statement :

$4x - 15 = 9$

$\Rightarrow 4x = 9 + 15$

$\Rightarrow 4x = 24$

$\Rightarrow X = 6$

Q 5. If Megha's age is increased by three times her age, the result is 60 years. Find her age

Solution :

Let Megha's age = x years

Three times Megha's age = 3x years

According to the statement

$X + 3x = 60$

$\Rightarrow 4x = 60$

$\Rightarrow x = 15$

Megha's age = 15 years

Q 6. 28 is 12 less than 4 times a number. Find the number

Solution :

Let the required number be x

4 times the number = 4x

12 less than 4 times the number = 4x – 12

According to the statement

$4x - 12 = 28$

$\Rightarrow 4x = 28 + 12$

$\Rightarrow 4x = 40$

$X = 10$

Required number = 10

Q 7. Five less than 3 times a number is -20. Find the number

Solution :

Let the required number = x

3 times the number = 3x

5 less than 3 times the number = $3x - 5$

According to statement :

$3x - 5 = - 20$

$\Rightarrow 3x = -20 + 5$

$\Rightarrow 3x = - 15$

$\Rightarrow x = -5$

Required number = - 5

Q 8. After 12 years, I shall be 3 times as old as 1 was 4 years ago. Find my present age.

Solution :

Let present age = x years

According to question,

$(x + 12) = 3 (x - 4)$

$(x + 12 = 3x - 12$

$2x = 24$

$\Rightarrow X = 12$ years

Present age = 12 years

Q 9. A man sold an article for 7396 and gained 10% on it. Find the cost price of the article.

Solution :

S.P. of article = Rs. 396

Gain = 10%

Let cost price = Rs. X

$$\therefore \text{ S.P.} = \frac{x \times (100+10)}{100} = \frac{110}{100}x$$

$$\therefore \frac{110}{100}x = 396$$

$$\Rightarrow x = \frac{396 \times 100}{110} = 360$$

Cost price of an article = Rs. 360

Q 10. The sum of two numbers is 4500,

if 10% of one number is 12.5% of the other, find the numbers.

Solution :

Let the first number = x

And the second number = y

According to question,

x + y = 4500 ……………… (i)

and 10% x = 12.5% y

i.e. 10x = 12.5y

$$x = \frac{12.5}{10}y \qquad ...(ii)$$

Substitute the value of x in equation (i),

$$\frac{12.5}{10}y + y = 45{,}000$$

$$12.5y + 10y = 45{,}000$$

$$22.5y = 45{,}000$$

$$y = \frac{45{,}000}{22.5} = 2000$$

Now, put the value of y in equation (ii)

$$x = \frac{12.5}{10} \times 2000$$

x = 2500

Hence, the numbers are 2500 and 2000.

Linear Inequations

[Chapter – 16]

1 If the replacement set is the set of natural numbers (N), find the solution set of : (i) $3x + 4 < 16$ (ii) $8 - x \leq 4x - 2$.

Solution :

(i) $3x + 4 < 16$ $\Rightarrow 3x < 16 - 4$ [Using rule 1]

$\Rightarrow 3x < 12$

$\Rightarrow \frac{3x}{3} < \frac{12}{3}$ [Using rule 3]

i.e. $x < 4$

Since, the replacement set = N (set of natural numbers)

$\therefore$ **Solution set = {1, 2, 3}** **Ans.**

(ii) $8 - x \leq 4x - 2$ $\Rightarrow -x - 4x \leq -2 - 8$ [Using rule 1]

$\Rightarrow -5x \leq -10$

$\Rightarrow \frac{-5x}{-5} \geq \frac{-10}{-5}$ [Using rule 4]

i.e. $x \geq 2$

Since, the replacement set = N

$\therefore$ **Solution set = {2, 3, 4, 5, 6, ...}**

Alternative method :

$8 - x \leq 4x - 2$ $\Rightarrow 4x - 2 \geq 8 - x$ [$x \leq y$ and $y \geq x$ mean the same]

$\Rightarrow 4x + x \geq 8 + 2$ [Using rule 2]

$\Rightarrow 5x \geq 10$

$\Rightarrow x \geq 2$ and $x \in N$

$\therefore$ **Solution set = {2, 3, 4, 5, 6, ...}** **Ans.**

2 If the replacement set is the set of whole numbers (W), find the solution set of : (i) $5x + 4 \le 24$ (ii) $4x - 2 < 2x + 10$.

Solution :

(i) $5x + 4 \le 24 \Rightarrow 5x \le 24 - 4$

$\Rightarrow 5x \le 20$ and $x \le \frac{20}{5}$ *i.e.* $x \le 4$

Since, the replacement set is the set of whole numbers

$\therefore$ Solution set = {0, 1, 2, 3, 4} **Ans.**

(ii) $4x - 2 < 2x + 10 \Rightarrow 4x - 2x < 10 + 2$

$\Rightarrow 2x < 12$ and $x < 6$

Since, the replacement set = W(whole numbers)

$\therefore$ Solution set = {0, 1, 2, 3, 4, 5} **Ans.**

3 If the replacement set is the set of integers, (I or Z), between −6 and 8, find the solution set of : (i) $6x - 1 \ge 9 + x$ (ii) $15 - 3x > x - 3$.

Solution :

(i) $6x - 1 \ge 9 + x \Rightarrow 6x - x \ge 9 + 1$

$\Rightarrow 5x \ge 10$ and $x \ge 2$

Since, the replacement set is the set of integers between −6 and 8.

$\therefore$ Solution set = {2, 3, 4, 5, 6, 7} **Ans.**

(ii) $15 - 3x > x - 3 \Rightarrow -3x - x > -3 - 15$

$\Rightarrow -4x > -18$

$\Rightarrow \frac{-4x}{-4} < \frac{-18}{-4}$ [Using rule 4]

$\Rightarrow x < 4{\cdot}5$

Since, the replacement set is the set of integers between −6 and 8

$\therefore$ Solution set = {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4} **Ans.**

Alternative method :

$15 - 3x > x - 3 \Rightarrow x - 3 < 15 - 3x$

$\Rightarrow x + 3x < 15 + 3$

$\Rightarrow 4x < 18$ and $x < 4{\cdot}5$

$\therefore$ Solution set = {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4} **Ans.**

4 If the replacement set is the set of real numbers (R), find the solution set of : (i) $5 - 3x < 11$ (ii) $8 + 3x \ge 28 - 2x$.

Solution :

(i) $5 - 3x < 11 \Rightarrow -3x < 11 - 5$

$$\Rightarrow \quad -3x < 6$$

$$\Rightarrow \quad \frac{-3x}{-3} > \frac{6}{-3}$$ [Using rule 4]

$$\Rightarrow \quad x > -2$$

Since, the replacement set is the set of real numbers (R)

∴ **Solution set = $\{x : x > -2$ and $x \in R\}$** **Ans.**

(ii) $8 + 3x \geq 28 - 2x \quad \Rightarrow \quad 3x + 2x \geq 28 - 8$

$$\Rightarrow \quad 5x \geq 20 \text{ and } x \geq 4$$

Since, the replacement set is the set of real numbers

∴ **Solution set = $\{x : x \geq 4$ and $x \in R\}$** **Ans.**

5 Solve : $\frac{x}{2} - 5 \leq \frac{x}{3} - 4$, where x is a positive odd integer.

Solution :

$$\frac{x}{2} - 5 \leq \frac{x}{3} - 4 \quad \Rightarrow \quad \frac{x}{2} - \frac{x}{3} \leq -4 + 5$$

$$\Rightarrow \quad \frac{3x - 2x}{6} \leq 1 \quad \Rightarrow \quad x \leq 6$$

Since, x is a positive odd integer

∴ **Solution set = {1, 3, 5}** **Ans.**

6 Solve the following inequation : $2y - 3 < y + 1 \leq 4y + 7$; if :
(i) $y \in$ {Integers} (ii) $y \in$ R (real numbers)

Solution :

$$2y - 3 < y + 1 \leq 4y + 7 \quad \Rightarrow \quad 2y - 3 < y + 1 \quad \text{and} \quad y + 1 \leq 4y + 7$$

$$\Rightarrow \quad y < 4 \quad \text{and} \quad -6 \leq 3y$$

$$\Rightarrow \quad y < 4 \quad \text{and} \quad y \geq -2$$

$$\Rightarrow \quad -2 \leq y < 4$$

(i) **When $y \in$ {Integers}**

∴ **Solution set = {–2, –1, 0, 1, 2, 3}** **Ans.**

(ii) **When $y \in$ R (real numbers)**

∴ **Solution set = $\{y : -2 \leq y < 4$ and $y \in R\}$** **Ans.**

Rational Numbers

[Chapter – 1]

Q 1. Add each pair rational numbers given below and
Show that their addition (sum) is also a rational number:

(i) $\frac{-5}{8}$ and $\frac{3}{8}$ (ii) $\frac{-8}{13}$ and $\frac{-4}{13}$

(iii) $\frac{6}{11}$ and $\frac{-9}{11}$ (iv) $\frac{5}{-26}$ and $\frac{8}{39}$

(v) $\frac{5}{-6}$ and $\frac{2}{3}$ (vi) -2 and $\frac{2}{5}$

(vii) $\frac{9}{-4}$ and $\frac{-3}{8}$ (viii) $\frac{7}{-18}$ and $\frac{8}{27}$

Solution:

(i) $\frac{-5}{8}$ and $\frac{3}{8}$

$= \frac{-5}{8} + \frac{3}{8}$

($\because$ Denominators are same, $\therefore$ LCM = 8)

$= \frac{-5+3}{8}$

$= \frac{-2}{8} = \frac{-1}{4}$

Which is a rational number.

(iv) $\frac{5}{-26}$ and $\frac{8}{39}$

$= \frac{5}{-26} + \frac{8}{39}$

$= \frac{-5\times3}{26\times3} + \frac{8\times2}{39\times2}$

2	26, 39
3	13, 39
13	13, 13
	1, 1

$\therefore$ LCM of 26 and 39 = 2 × 3 × 13 = 78

$= \frac{-15+16}{78}$ ($\because$ LCM of 26 and 39 = 78)

$= \frac{1}{78}$

Which is a rational number.

(*v*) $\frac{5}{-6}$ and $\frac{2}{3}$

$= \frac{-5}{6} + \frac{2}{3}$

$$\begin{array}{c|c} 2 & 6, 3 \\ \hline 3 & 3, 3 \\ \hline & 1, 1 \end{array}$$

$\therefore$ LCM of 6, 3 = 2 × 3 = 6

$= \frac{-5 \times 1}{6 \times 1} + \frac{2 \times 2}{3 \times 2}$

($\because$ LCM of 6 and 3 = 6)

$= \frac{-5+4}{6} = \frac{-1}{6}$

(*vi*) –2 and $\frac{2}{5}$

$= \frac{-2}{1} + \frac{2}{5}$ ($\because$ LCM of 1 and 5 = 5)

$= \frac{-2 \times 5}{1 \times 5} + \frac{2 \times 1}{5 \times 1}$

$= \frac{-10+2}{5} = \frac{-8}{5}$

Which is a rational number.

(*vii*) $\frac{9}{-4}$ and $\frac{-3}{8}$

$= \frac{-9}{4} + \left(\frac{-3}{8}\right)$

$$\begin{array}{c|c} 2 & 4, 8 \\ \hline 2 & 2, 4 \\ \hline 2 & 2, 2 \\ \hline & 1, 1 \end{array}$$

$\therefore$ LCM of 4 and 8 = 2 × 2 × 2 = 8

$= \frac{-9 \times 2}{4 \times 2} - \frac{3 \times 1}{8 \times 1}$

($\because$ LCM of 4 and 8 = 8)

$= \frac{-18-3}{8} = \frac{-21}{8}$

Which is a rational number.

(*viii*) $\frac{7}{-18}$ and $\frac{8}{27}$

$\frac{7}{-18} + \frac{8}{27}$

$= \frac{-7\times3}{18\times3} + \frac{8\times2}{27\times2}$

2	18, 27
3	9, 27
3	3, 9
3	1, 3
	1, 1

$\therefore$ LCM of 18 and 27 = $2\times3\times3\times3 = 54$

$= \frac{-21+16}{54} = \frac{-5}{54}$

Which is a rational number.

Q 2. **Evaluate:**

(*i*) $\frac{5}{9} + \frac{-7}{6}$ (*ii*) $4 + \frac{3}{-5}$

(*iii*) $\frac{1}{-15} + \frac{5}{-12}$ (*iv*) $\frac{5}{9} + \frac{3}{-4}$

(*v*) $\frac{-8}{9} + \frac{-5}{12}$ (*vi*) $0 + \frac{-2}{7}$

(*vii*) $\frac{5}{-11} + 0$ (*viii*) $2 + \frac{-3}{5}$

(*ix*) $\frac{4}{-9} + 1$

Solution:

(*i*) $\frac{5}{9} + \frac{-7}{6}$

2	9, 6
3	9, 3
3	3, 1
	1, 1

$\therefore$ LCM of 9 and 6 = $2\times3\times3 = 18$

$= \frac{5\times2}{9\times2} - \frac{7\times3}{6\times3}$

($\because$ LCM of 9 and 6 = 18)

$= \frac{10-21}{18} = \frac{-11}{8}$

(*ii*) $4 + \frac{3}{-5}$

$= \frac{4}{1} + \left(\frac{3}{-5}\right)$

$= \frac{4}{1} - \frac{3}{5}$

$= \frac{4\times5}{1\times5} - \frac{3\times1}{5\times1}$ ($\because$ LCM of 1 and 5 = 5)

$= \frac{20-3}{5} = \frac{17}{5} = 3\frac{2}{5}$

(*iii*) $\frac{1}{-15} + \frac{5}{-12}$

$= \frac{-1}{15} + \left(\frac{5}{-12}\right)$

$= \frac{-1}{15} - \frac{5}{12}$

2	15, 12
2	15, 6
3	15, 3
5	5, 1
	1, 1

$\therefore$ LCM of 15 and 12 = $2\times2\times3\times5 = 60$

$= \frac{-1\times4}{15\times4} - \frac{5\times5}{12\times5}$

($\because$ LCM of 15 and 12 = 60)

$= \frac{-4-25}{60} = \frac{-29}{60}$

(iv) $\frac{5}{9} + \frac{3}{-4}$

$= \frac{5}{9} - \frac{3}{4}$

2	9, 4
2	9, 2
3	9, 1
3	3, 1
	1, 1

($\therefore$ LCM of 9 and 4 = 2 × 2 × 3 × 3 = 36)

$= \frac{5\times4}{9\times4} - \frac{3\times9}{4\times9}$

$= \frac{20-27}{36} = \frac{-7}{36}$

($\because$ LCM of 9 and 4 = 36)

$= \frac{-7}{36}$

(v) $\frac{-8}{9} + \frac{-5}{12}$

2	9, 12
2	9, 6
3	9, 3
3	3, 1
	1, 1

$\therefore$ LCM of 9, 12 = 2 × 2 × 3 × 3 = 36

$= \frac{-8\times4}{9\times4} - \frac{5\times3}{12\times3}$

$= \frac{-32-15}{36}$ ($\because$ LCM of 9 and 12 = 36)

$= \frac{-47}{36}$

(vi) $0 + \frac{-2}{7}$

$= \frac{0\times7}{1\times7} - \frac{2\times1}{7\times1}$ ($\because$ LCM of 0 and 7 = 7)

$= \frac{0-2}{7} = \frac{-2}{7}$

(vii) $\frac{5}{-11} + 0$

$= \frac{-5\times1}{11\times1} + \frac{0\times11}{1\times11}$

($\because$ LCM of 0 and 11 = 11)

(ix) $\frac{4}{-9} + 1$

$= \frac{-4}{9} + \frac{1}{1}$

$= \frac{-4\times1}{9\times1} +$

$= \frac{-4+9}{9} = \frac{5}{9}$

Q 3. **Evaluate:**

(i) $\frac{2}{3} - \frac{4}{5}$ (ii) $\frac{-4}{9} - \frac{2}{-3}$

(iii) $-1 - \frac{4}{9}$ (iv) $\frac{-2}{7} - \frac{3}{-14}$

(v) $\frac{-5}{18} - \frac{-2}{9}$ (vi) $\frac{5}{21} - \frac{-13}{42}$

Solution:

(i) $\frac{2}{3} - \frac{4}{5}$

3	3, 5
5	1, 5
	1, 1

$\therefore$ LCM of 3 and 5 = 15

$= \frac{2\times5}{3\times5} - \frac{4\times3}{5\times3}$ ($\because$ LCM of 3 and 5 = 15)

$= \frac{10-12}{15} = \frac{-2}{15}$

(*ii*) $\frac{-4}{9} - \frac{2}{-3}$

$$\begin{array}{c|l} 3 & 9, 3 \\ \hline 5 & 3, 1 \\ \hline & 1, 1 \end{array}$$

$= \frac{-4 \times 1}{9 \times 1} - \frac{(-2 \times 3)}{3 \times 3}$

($\because$ LCM of 3 and 9 = 9)

$= \frac{-4+6}{9} = \frac{2}{9}$

(*iii*) $-1 - \frac{4}{9}$

$= \frac{-1 \times 9}{1 \times 9} - \frac{4 \times 1}{9 \times 1}$

$= \frac{-9-4}{9} = \frac{-13}{9}$

(*iv*) $\frac{-2}{7} - \frac{3}{-14}$

$$\begin{array}{c|l} 2 & 7, 14 \\ \hline 7 & 7, 7 \\ \hline & 1, 1 \end{array}$$

$\therefore$ LCM of 7 and 14 = 14

$= \frac{-2 \times 2}{7 \times 2} - \frac{(-3 \times 1)}{14 \times 1}$

($\because$ LCM of 7 and 14 = 14)

$= \frac{-4+3}{14} = \frac{-1}{14}$

(*v*) $\frac{-5}{18} - \frac{-2}{9}$

$$\begin{array}{c|l} 2 & 18, 9 \\ \hline 2 & 6, 9 \\ \hline 3 & 3, 9 \\ \hline 3 & 1, 3 \\ \hline & 1, 1 \end{array}$$

$\therefore$ LCM of 9 and 18 = $2 \times 2 \times 3 \times 3 = 36$

$= \frac{-5 \times 2}{18 \times 2} - \frac{(-2 \times 4)}{9 \times 4}$

($\because$ LCM of 18 and 9 = 36)

$= \frac{-10+8}{36}$

$= \frac{-2}{36} = \frac{-1}{18}$

(*vi*) $\frac{5}{21} - \frac{-13}{42}$

$$\begin{array}{c|l} 2 & 21, 42 \\ \hline 3 & 21, 21 \\ \hline 7 & 7, 7 \\ \hline & 1, 1 \end{array}$$

$\therefore$ LCM of 21, 42 = $2 \times 3 \times 7 = 42$

$= \frac{5 \times 2}{21 \times 2} - \frac{(-13 \times 1)}{42 \times 1}$

($\because$ LCM of 21 and 42 = 42)

$= \frac{10+13}{42} = \frac{23}{42}$

Q 4. **Subtract:**

(i) $\frac{5}{8}$ from $\frac{-3}{8}$ (ii) $\frac{-8}{11}$ from $\frac{4}{11}$

(iii) $\frac{4}{9}$ from $\frac{-5}{9}$ (iv) $\frac{1}{4}$ from $\frac{-3}{8}$

(v) $\frac{-5}{8}$ from $\frac{-13}{16}$ (vi) $\frac{-9}{22}$ from $\frac{5}{33}$

Solution:

(i) $\frac{5}{8}$ from $\frac{-3}{8}$

$= \frac{-3}{8} - \frac{5}{8}$

$= \frac{-3\times1}{8\times1} - \frac{5\times1}{8\times1}$

$= \frac{-3-5}{8} = \frac{-8}{8} = -1$

(ii) $\frac{-8}{11}$ from $\frac{4}{11}$

$= \frac{4}{11} - \left(\frac{-8}{11}\right)$

$= \frac{4+8}{11} = \frac{12}{11} = 1\frac{1}{11}$

(iii) $\frac{4}{9}$ from $\frac{-5}{9}$

$= \frac{-5}{9} - \frac{4}{9}$

$= \frac{-5-4}{9} = \frac{-9}{9} = -1$

(iv) $\frac{1}{4}$ from $\frac{-3}{8}$

$$\begin{array}{r|l} 2 & 4, 8 \\ 2 & 2, 4 \\ 2 & 1, 2 \\ & 1, 1 \end{array}$$

$\therefore$ LCM of 4, 8 $= 2 \times 2 \times 2 = 8$

$= \frac{-3}{8} - \frac{1}{4}$ ($\because$ LCM of 8 and 4 = 8)

$= \frac{-3\times1}{8\times1} - \frac{1\times2}{4\times2}$

$= \frac{-3-2}{8} = \frac{-5}{8}$

(v) $\frac{-5}{8}$ from $\frac{-13}{16}$

$$\begin{array}{r|l} 2 & 8, 16 \\ 2 & 4, 8 \\ 2 & 2, 4 \\ 2 & 1, 2 \\ & 1, 1 \end{array}$$

$\therefore$ LCM of 8 and 16 = 16

$= \frac{-13}{16} - \left(\frac{-5}{8}\right)$

$= \frac{-13\times1}{16\times1} + \frac{5\times2}{8\times2}$

($\because$ LCM of 8 and 16 = 16)

$= \frac{-13+10}{16} = \frac{-3}{16}$

(vi) $\frac{-9}{22}$ from $\frac{5}{33}$

$$\begin{array}{r|l} 2 & 22, 33 \\ 3 & 11, 33 \\ 11 & 1, 11 \\ & 1, 1 \end{array}$$

$\therefore$ LCM of 22 and 33 $= 2 \times 3 \times 11 = 66$

$= \frac{5}{33} - \left(\frac{-9}{22}\right)$

$= \frac{5\times2}{33\times2} + \frac{9\times3}{22\times3}$

($\because$ LCM of 22 and 33 = 66)

$= \frac{10+27}{66} = \frac{37}{66}$

Q 5. Evaluate:

(i) $\frac{-14}{5} \times \frac{-6}{7}$ (ii) $\frac{7}{6} \times \frac{-18}{91}$

(iii) $\frac{-125}{72} \times \frac{9}{-5}$ (iv) $\frac{-11}{9} \times \frac{-51}{-44}$

(v) $-\frac{16}{5} \times \frac{20}{8}$

Solution:

(i) $\frac{-14}{5} \times \frac{-6}{7}$

$= \frac{(-14)\times(-6)}{5\times7} = \frac{(-2)\times(-6)}{5\times1}$

$= \frac{12}{5} = 2\frac{2}{5}$

(ii) $\frac{7}{6} \times \frac{-18}{91}$

$= \frac{7\times(-18)}{6\times91} = \frac{1\times(-3)}{1\times13} = \frac{-3}{13}$

(iii) $\frac{-125}{72} \times \frac{9}{-5}$

$= \frac{(-125)\times9}{72\times-5} = \frac{25\times1}{8\times1}$

25 1

(iv) $\frac{-11}{9} \times \frac{-51}{-44}$

$= \frac{(-11)\times(-51)}{9\times(-44)} = \frac{1\times(-51)}{9\times4}$

$= \frac{-51}{36} = \frac{-17}{12}$

(v) $-\frac{16}{5} \times \frac{20}{8}$

$= \frac{(-16)\times20}{5\times8} = \frac{(-2)\times4}{1\times1} = -8$

Q 6. **Multiply:**

(i) $\frac{5}{6}$ and $\frac{8}{9}$ (ii) $\frac{2}{7}$ and $\frac{-14}{9}$

(iii) $\frac{-7}{8}$ and 4 (iv) $\frac{36}{-7}$ and $\frac{-9}{28}$

(v) $\frac{-7}{10}$ and $\frac{-8}{15}$ (vi) $\frac{3}{-2}$ and $\frac{-7}{3}$

Solution: (i) $\frac{5}{6}$ and $\frac{8}{9}$

$= \frac{5\times8}{6\times9} = \frac{5\times4}{3\times9} = \frac{20}{27}$

(ii) $\frac{2}{7}$ and $\frac{-14}{9}$

$= \frac{2\times(-14)}{7\times9} = \frac{2\times(-2)}{1\times9} = \frac{-4}{9}$

(iii) $\frac{-7}{8}$ and 4

$= \frac{(-7)\times4}{8\times1} = \frac{(-7)\times1}{2\times1} = \frac{-7}{2} = 3\frac{1}{2}$

(iv) $\frac{36}{-7}$ and $\frac{-9}{28}$

$= \frac{36\times(-9)}{(-7)\times28} = \frac{9\times(-9)}{(-7)\times7}$

$= \frac{-81}{-49} = \frac{81}{49} = 1\frac{32}{49}$

(v) $\frac{-7}{10}$ and $\frac{-8}{15}$

$= \frac{(-7)\times(-8)}{10\times15} = \frac{(-7)\times(-4)}{5\times15} = \frac{28}{75}$

(vi) $\frac{3}{-2}$ and $\frac{-7}{3}$

$= \frac{3\times(-7)}{(-2)\times3} = \frac{1\times(-7)}{(-2)\times1}$

$= \frac{-7}{-2} = \frac{7}{2} = 3\frac{1}{2}$

Q 7. **Evaluate:**

(i) $1 \div \frac{1}{3}$ (ii) $3 \div \frac{3}{5}$

(iii) $-\frac{5}{12} \div \frac{1}{16}$ (iv) $-\frac{21}{16} \div \left(\frac{-7}{8}\right)$

(v) $0 \div \left(-\frac{4}{7}\right)$ (vi) $\frac{8}{-5} \div \frac{24}{25}$

(vii) $-\frac{3}{4} \div (-9)$ (viii) $\frac{3}{4} \div \left(-\frac{5}{12}\right)$

(ix) $-5 \div \left(-\frac{10}{11}\right)$ (x) $\frac{-7}{11} \div \left(\frac{-3}{44}\right)$

Solution:

(i) $1 \div \frac{1}{3}$

$= 1 \times \frac{3}{1} = 3$

(ii) $3 \div \frac{3}{5}$

$= 3 \times \frac{5}{3} = \frac{1 \times 5}{1 \times 1} = 5$

(iii) $-\frac{5}{12} \div \frac{1}{16}$

$= -\frac{5}{12} \times \frac{16}{1}$

$= \frac{-5 \times 4}{3 \times 1} = \frac{-20}{3} = -5\frac{5}{3}$

(iv) $-\frac{21}{16} \div \left(\frac{-7}{8}\right)$

$= -\frac{21}{16} \times \frac{8}{-7}$

$= \frac{3 \times 1}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$

(v) $0 \div \left(-\frac{4}{7}\right)$

$= 0 \times \left(-\frac{7}{4}\right) = 0$

(vi) $\frac{8}{-5} \div \frac{24}{25}$

$= \frac{8}{-5} \times \frac{25}{24}$

$= \frac{2 \times 5}{(-1) \times 6} = \frac{1 \times 5}{(-1) \times 3} = \frac{-5}{3}$

(vii) $-\frac{3}{4} \div (-9)$

$= -\frac{3}{4} \times \frac{1}{-9} = \frac{(-1) \times 1}{4 \times (-3)} = \frac{1}{12}$

(viii) $\frac{3}{4} \div \left(-\frac{5}{12}\right)$

$= \frac{3}{4} \times \left(-\frac{12}{5}\right)$

$= \frac{3 \times 3}{1 \times (-5)} = -\frac{9}{5}$

(ix) $-5 \div \left(-\frac{10}{11}\right)$

$= -5 \times \frac{11}{-10}$

$= \frac{1 \times 11}{1 \times 2} = \frac{11}{2} = 5\frac{1}{2}$

(x) $\frac{-7}{11} \div \left(\frac{-3}{44}\right)$

$= \frac{-7}{11} \times \left(\frac{44}{-3}\right)$

$= \frac{(-7)\times 4}{1\times(-3)} = \frac{28}{3} = 9\frac{1}{3}$

Q 8. **Divide:**

(i) 3 by $\frac{1}{3}$ (ii) -2 by $\left(-\frac{1}{2}\right)$

(iii) 0 by $\frac{7}{-9}$ (iv) $\frac{-5}{8}$ by $\frac{1}{4}$

(v) $-\frac{3}{4}$ by $-\frac{9}{16}$

Solution: (i) 3 by $\frac{1}{3}$

$= 3 \div \frac{1}{3} = 3 \times \frac{3}{1} = 9$

(ii) -2 by $\left(-\frac{1}{2}\right)$

$= -2 \div \left(-\frac{1}{2}\right)$

$= -2 \times \frac{2}{-1} = 4$

(iii) 0 by $\frac{7}{-9}$

$= 0 \div \frac{7}{-9}$

$= 0 \times \frac{-9}{7} = 0$

(iv) $\frac{-5}{8}$ by $\frac{1}{4}$

$= \frac{-5}{8} \div \frac{1}{4}$

$= \frac{-5}{8} \times \frac{4}{1}$

$= \frac{-5\times 1}{2\times 1} = \frac{-5}{2}$

(v) $-\frac{3}{4}$ by $-\frac{9}{16}$

$= -\frac{3}{4} \div -\frac{9}{16}$

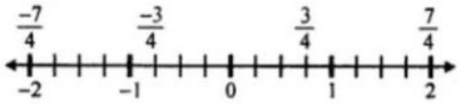
$= -\frac{3}{4} \times -\frac{16}{9} = \frac{(-1)\times 4}{1\times(-3)}$

$= \frac{-4}{-3} = \frac{4}{3} = 1\frac{1}{3}$

Q 9. Draw a number line and mark $\frac{3}{4}, \frac{7}{4}, \frac{-3}{4}$ and $\frac{-7}{4}$ on it.

Solution:

Draw a number line as shown below:



Q 10. Insert one rational number between

i. 0.7 and 8
ii. 3.5 and 5
iii. 2 and 3.2
iv. 4.2 and 3.6
v. $\frac{1}{2}$ and 2

Solution :

(*i*) The rational number between 7 and 8

$$= \frac{7+8}{2} = \frac{15}{2} = 7.5$$

(*ii*) The rational number between 3.5 and 5

$$= \frac{3.5+5}{2} = \frac{8.5}{2} = 4.25$$

(*iii*) The rational number between 2 and 3.2

$$= \frac{2+3.2}{2} = \frac{5.2}{2} = 2.6$$

(*iv*) The rational number between 4.2 and 3.6

$$= \frac{4.2+3.6}{2} = \frac{7.8}{2} = 3.9$$

(*v*) The rational number between $\frac{1}{2}$ and 2

$$= \frac{1+2}{2\times 2} = \frac{3}{4} = 1.25$$

Exponents and powers

[Chapter – 2]

1) Simplify

a) $\frac{(-5)^{21}}{(-5)^{25}}$

Ans. $\frac{(-5)^{21}}{(-5)^{25}} = \frac{1}{(-5)^{25-21}} = \frac{1}{(-5)^{4}} = \frac{1}{625}$

b) $\{(-5)^2\}^3$

Ans. $\{(-5)^2\}^3 = (-5)^6 = 15625$

2) Simplify

$$\frac{16 \times 10^4 \times 3^3}{6^5 \times 5^6}$$

Ans. $\frac{2^4 \times (2^4 \times 5^4) \times 3^3}{2^5 \times 3^5 \times 5^6}$

$$= \frac{2^{4+4} \times 3^5 \times 5^4}{2^5 \times 3^5 \times 5^6} = \frac{2^8 \times 3^3 \times 5^4}{2^5 \times 3^5 \times 5^6} = \frac{2^3}{3^2 \times 5^2} = \frac{8}{225}$$

3) Simplify

$$\left(\frac{3}{4}\right)^2 \times \left(\frac{-2}{5}\right)^3 \times 4^3$$

$$= \frac{3^2}{(2^2)^2} \times \frac{(-2)^3}{5^3} \times (2^2)^3$$

$$= \left\{\frac{3^2}{2^4} \times \frac{(+2)^3}{5^3} \times 2^6\right\}$$

$$= \left\{\frac{3^2 \times 2^{3+6}}{2^4 \times 5^3}\right\} = -\left\{\frac{3^2 \times 2^5}{5^3}\right\} = -\frac{288}{125} = -2\frac{38}{125}$$

Q 4.

Evaluate:

(*i*) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

(*ii*) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

(*iii*) $(2^{-1} + 3^{-1})^{3}$

(*iv*) $(3^{-1} \div 4^{-1})^{2}$

(*v*) $(2^{2} + 3^{2}) \times \left(\frac{1}{2}\right)^{2}$

(*vi*) $(5^{2} - 3^{2}) \times \left(\frac{2}{3}\right)^{-3}$

(*vii*) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

(*viii*) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^{2}$

(*ix*) $\left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2}$

(*x*) $(5^{-1} \times 3^{-1}) \div 6^{-1}$

Solution:

(*i*) $(3^{-1} \times 9^{-1}) \div 3^{-2}$

$= \left(\frac{1}{3} \times \frac{1}{9}\right) \div \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{27} \div \frac{1}{9}$

$= \frac{1}{27} \times \frac{9}{1} = \frac{1}{3}$

(ii) $(3^{-1} \times 4^{-1}) \div 6^{-1}$

$$= \left(\frac{1}{3} \times \frac{1}{4}\right) \div \frac{1}{6}$$

$$= \frac{1}{12} \div \frac{1}{6}$$

$$= \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$$

(iii) $(2^{-1} + 3^{-1})^3$

$$= \left(\frac{1}{2} + \frac{1}{3}\right)^3 = \left(\frac{1 \times 3}{2 \times 3} + \frac{1 \times 2}{3 \times 2}\right)^3$$

$$= \left(\frac{3+2}{6}\right)^3 = \left(\frac{5}{6}\right)^3$$

$$= \frac{5 \times 5 \times 5}{6 \times 6 \times 6} = \frac{125}{216}$$

(iv) $(3^{-1} \div 4^{-1})^2$

$$= \left(\frac{1}{3} \div \frac{1}{4}\right)^2$$

$$= \left(\frac{1}{3} \times \frac{4}{1}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9} = 1\frac{7}{9}$$

(v) $(2^2 + 3^2) \times \left(\frac{1}{2}\right)^2$

$$= (2 \times 2) + (3 \times 3) \times \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= 4 + 9 \times \frac{1}{4} = \frac{13}{4} = 3\frac{1}{4}$$

(vi) $(5^2 - 3^2) \times \left(\frac{2}{3}\right)^{-3}$

$$= (5 \times 5) - (3 \times 3) \times \left(\frac{3}{2}\right)^3$$

$$= 25 - 9 \times \left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}\right)$$

$$= 16 \times \frac{27}{8} = 54$$

(vii) $\left[\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{6}\right)^{-3}$

$$= \left[\left(\frac{4}{1}\right)^3 - \left(\frac{3}{1}\right)^3\right] \div \left(\frac{6}{1}\right)^3$$

$$= \left(\frac{4}{1} \times \frac{4}{1} \times \frac{4}{1} - \frac{3}{1} \times \frac{3}{1} \times \frac{3}{1}\right) \div \left(\frac{6}{1}\right)^3$$

$$= 64 - 27 \times \left(\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}\right)$$

$$= 37 \times \frac{1}{216} = \frac{37}{216}$$

(viii) $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-2\times 2} = \left(-\frac{3}{4}\right)^{-4}$

$$= \left(\frac{4}{3}\right)^4 = \frac{4 \times 4 \times 4 \times 4}{3 \times 3 \times 3 \times 3}$$

$$= \frac{256}{81} = 3\frac{13}{81}$$

$$(ix)\ \left\{\left(\frac{3}{5}\right)^{-2}\right\}^{-2} = \left(\frac{3}{5}\right)^{-2\times(-2)} = \left(\frac{3}{5}\right)^{4}$$

$$= \frac{3\times3\times3\times3}{5\times5\times5\times5} = \frac{81}{625}$$

$$(x)\ (5^{-1}\times3^{-1}) \div 6^{-1}$$

$$= \left(\frac{1}{5}\times\frac{1}{3}\right) \div \frac{1}{6}$$

$$= \frac{1}{15} \div \frac{1}{6}$$

$$= \frac{1}{15} \times \frac{6}{1} = \frac{2}{5}$$

Q 5.

Simplify:

$$(i)\ 8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$$

$$(ii)\ [(64)^{-2}]^{-3} + [\{(-8)^{2}\}^{3}]^{2}$$

$$(iii)\ (2^{-3} - 2^{-4})\ (2^{-3} + 2^{-4})$$

Solution:

$$(i)\ 8^{\frac{4}{3}} + 25^{\frac{3}{2}} - \left(\frac{1}{27}\right)^{-\frac{2}{3}}$$

$$= (2^{3})^{\frac{4}{3}} + (5^{2})^{\frac{3}{2}} - \left(\frac{1}{3^{3}}\right)^{-\frac{2}{3}}$$

$$= 2^{3\times\frac{4}{3}} + 5^{2\times\frac{3}{2}} - \frac{1}{3^{3\times\left(\frac{-2}{3}\right)}}$$

$$= 2^{4} + 5^{3} - \frac{1}{3^{-2}}$$

$$= 16 + 125 - 3^{2}$$

$$= 141 - 9 = 132$$

$$= 2^{3\times\frac{4}{3}} + 5^{2\times\frac{3}{2}} - \frac{1}{3^{3\times\left(\frac{-2}{3}\right)}}$$

$$= 2^{4} + 5^{3} - \frac{1}{3^{-2}}$$

$$= 16 + 125 - 3^{2}$$

$$= 141 - 9 = 132$$

(*ii*) $[(64)^{-2}]^{-3} + [\{(-8)^2\}^3]^2$

$= (2^6)^{-2 \times -3} + (-8)^{2 \times 3 \times 2}$

$= 2^{6 \times (6)} + (-8)^{12}$

$= 2^{+36} \div (-8)^{12}$

$= 2^{+36} + [(-2)^3]^{12} = 2^{36} + (-2)^{36}$

$= \dfrac{2^{36}}{(-2)^{36}} = \dfrac{2^{36}}{2^{36}}$ (∵ 36 is even)

$= 2^{36-36} = 2^0 = 1$ (∵ $a^0 = 1$)

(*iii*) $(2^{-3} - 2^{-4})(2^{-3} + 2^{-4})$

$= (2^{-3})^2 - (2^{-4})^2$

$\{\because (a - b)(a + b) = a^2 - b^2\}$

$= 2^{-6} - 2^{-8} = \dfrac{1}{2^6} - \dfrac{1}{2^8}$

$= \dfrac{1}{64} - \dfrac{1}{256}$

$= \dfrac{4-1}{256} = \dfrac{3}{256}$

Q 6.

Evaluate:

(*i*) $(-5)^0$

(*ii*) $8^0 + 4^0 + 2^0$

(*iii*) $(8 + 4 + 2)^0$

(*iv*) $4x^0$

(*v*) $(4x)^0$

(*vi*) $[(10^3)^0]^5$

(*vii*) $(7x^0)^2$

(*viii*) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

Solution:

(i) $(-5)^0 = 1$ $\quad (\because a^0 = 1)$

(ii) $8^0 + 4^0 + 2^0$

$= 1 + 1 + 1 = 3$ $\quad (\because a^0 = 1)$

(iii) $(8 + 4 + 2)^0 = (14)^0 = 1$ $\quad (\because a^0 = 1)$

(iv) $4x^0 = 4 \times 1 = 4$

(v) $(4x)^0 = 1$

(vi) $[(10^3)^0]^5 = 10^{3 \times 0 \times 5} = 10^0 = 1$

(vii) $(7x^0)^2 = 7^2 \times x^{0 \times 2} = 49 \times 1 = 49$

(viii) $9^0 + 9^{-1} - 9^{-2} + 9^{\frac{1}{2}} - 9^{-\frac{1}{2}}$

$$= 1 + \frac{1}{9} - \frac{1}{9^2} + (3^2)^{\frac{1}{2}} - (3^2)^{-\frac{1}{2}}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3^{2 \times \frac{1}{2}} - 3^{2 \times \left(-\frac{1}{2}\right)}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + 3 - 3^{-1}$$

$$= 1 + \frac{1}{9} - \frac{1}{81} + \frac{3}{1} - \frac{1}{3}$$

$$= \frac{81 + 9 - 1 + 243 - 27}{81} = \frac{333 - 28}{81}$$

$$= \frac{305}{81} = 3\frac{62}{81}$$

Cubes and Cube Roots

[Chapter – 3]

Q 1

Find the square of :

(i) 59

(ii) 63

(iii) 15

Solution:

(i) Square of 59= 59 x 59 = 3481

(ii) Square of 6.3 = 6.3 x 6.3 = 39.69

(iii) Square of 15 = 15 x 15 = 225

Q 2.

By splitting into prime factors, find the square root of :

(i) 11025

(if) 396900

(iii) 194481

Solution:

(i) $\sqrt{11025}$

$= \sqrt{\overline{5\times5}\times\overline{7\times7}\times\overline{3\times3}}$

$= 5\times7\times3 = 105$

5	11025
5	2205
7	441
7	63
3	9
	3

(ii) $\sqrt{396900}$

$= \sqrt{\overline{2\times2}\times\overline{3\times3}\times\overline{3\times3}\times\overline{5\times5}\times\overline{7\times7}}$

$= 2\times3\times3\times5\times7 = 630$

2	396900
2	198450
3	99225
3	33075
3	11025
3	3675
5	1225

5	245
7	49
	7

(iii) $\sqrt{194481}$

$= \sqrt{\overline{3\times3}\times\overline{3\times3}\times\overline{7\times7}\times\overline{7\times7}}$

$= 3\times3\times7\times7 = 441$

3	194481
3	64827
3	21609
3	7203
7	2401
7	343
7	49
	7

Q 3.

Find the square root of:

(i) 4761

(ii) 7744

(iii) 15129

(iv) 0.2916

(v) 0.001225

(vi) 0.023104

(vii) 27.3529

Solution:

Sol. (*i*) 4761

	69
6	4761
	36
129	1161
	1161
	×

Required square root = 69

(*ii*) 7744

	88
8	7744
	64
168	1344
	1344
	×

Required square root = 88

(*iii*) 15129

	123
1	15129
	1
22	51
	44
243	729
	729
	×

Required square root = 123

(*iv*) 0.2916

	0.54
0.5	0.2916
	0.25
0.104	416
	416
	×

Required square root = 0.54

(*v*) 0.001225

	0.035
0.03	0.001225
	9
0.065	325
	325
	×

Required square root = 0.035

(*vi*) 0.023104

	0.152
0.1	0.023104
	0.01
.25	131
	125
.302	604
	604
	×

Required square root of = 0.152

(*vii*) 27.3529

	5.23
5	27.3529
	25
102	2.35
	2.04
1043	3129
	3129
	×

Required square root = 5.23

Q 4.

Find the square root of:

(i) 4.2025

(ii) 531.7636

(iii) 0.007225

Solution:

Sol. (*i*) 4.2025

	2.05
2	4.2025
	4
405	.2025
	.2025
	×

Required square root = 2.05

(*ii*) 531.7636

	23.06
2	531.7636
	4
43	131
	129
4606	2.7636
	2.7636
	×

Required square root = 23.06

(*iii*) 0.007225

	0.085
.8	.007225
	64
0.165	825
	825
	×

Required square root = 0.085

Q 5.

Find the cube of :

(i) 7

(ii) 11

(iii) 16

(iv) 23

(v) 31

(vi) 42

(vii) 54

Solution:

(i) $(7)^3 = 7 \times 7 \times 7 = 343$

(ii) $(11)^3 = 11 \times 11 \times 11 = 1331$

(iii) $(16)^3 = 16 \times 16 \times 16 = 4096$

(iv) $(23)^3 = 23 \times 23 \times 23 = 12167$

(v) $(31)^3 = 31 \times 31 \times 31 = 29791$

(vi) $(42)^3 = 42 \times 42 \times 42 = 74088$

(vii) $(54)^3 = 54 \times 54 \times 54 = 157464$

Q 6.

Find the cubes of :

(i) $\frac{3}{7}$

(ii) $\frac{8}{9}$

(iii) $\frac{10}{13}$

(iv) $1\frac{2}{7}$

(v) $2\frac{1}{2}$

Solution:

$$(i)\ \frac{3}{7} = \left(\frac{3}{7}\right)^3 = \frac{3\times3\times3}{7\times7\times7} = \frac{27}{343}$$

$$(ii)\ \frac{8}{9} = \left(\frac{8}{9}\right)^3 = \frac{8\times8\times8}{9\times9\times9} = \frac{512}{729}$$

$$(iii)\ \frac{10}{13} = \left(\frac{10}{13}\right)^3 = \frac{10\times10\times10}{13\times13\times13} = \frac{1000}{2197}$$

$$(iv)\ 1\frac{2}{7} = \left(1\frac{2}{7}\right)^3 = \left(\frac{1\times7+2}{7}\right)^3 = \left(\frac{9}{7}\right)^3$$

$$= \frac{9\times9\times9}{7\times7\times7} = \frac{729}{343} = 2\frac{43}{343}$$

$$(v)\ 2\frac{1}{2} = \left(2\frac{1}{2}\right)^3 = \left(\frac{5}{2}\right)^3$$

$$= \frac{5\times5\times5}{2\times2\times2} = \frac{125}{8} = 15\frac{5}{8}.$$

Q 7.

Find the cubes of :

(i) -3

(ii) -7

(iii) -12

(iv) -18

(v) -25

(vi) -30

(vii) -50

Solution:

(*i*) $-3 = (-3)^3 = -3 \times -3 \times -3$
$= -(3 \times 3 \times 3) = -27$

(*ii*) $-7 = (-7)^3 = -7 \times -7 \times -7$
$= -(7 \times 7 \times 7) = -343$

(*iii*) $-12 = (-12)^3 = \quad 12 \times -12 \times -12$
$= -(12 \times 12 \times 12) = -1728$

(*iv*) $-18 = (-18)^3 = -18 \times -18 \times -18$
$= -(18 \times 18 \times 18) = -5832$

(*v*) $-25 = (-25)^3 = -25 \times -25 \times -25$
$= -(25 \times 25 \times 25) = -15625$

(*vi*) $-30 = (-30)^3 = -30 \times -30 \times -30$
$= -(30 \times 30 \times 30) = -27000$

(*vii*) $-50 = (-50)^3 = -50 \times -50 \times -50$
$= -(50 \times 50 \times 50) = -125000$

Q 8.

Find the cube-roots of :

(i) 64

(ii) 343

(iii) 729

(iv) 1728

(v) 9261

(vi) 4096

(vii) 8000

(viii) 3375

Solution:

(*i*) $64 = \sqrt[3]{64} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 = 4$

2	64
2	32
2	16
2	8
2	4
2	2
	1

(*ii*) $343 = \sqrt[3]{343} = 7 \times 7 \times 7 = 7$

7	343
7	49
7	7
	1

(*iii*) $729 = \sqrt[3]{729} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$= 3 \times 3 = 9$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(*iv*) $1728 = \sqrt[3]{1728} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\times (3 \times 3 \times 3)$

$= 2 \times 2 \times 3 = 12$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(*v*) 9261 = $\sqrt[3]{9261} = (3 \times 3 \times 3) \times (7 \times 7 \times 7)$

$= 3 \times 7 = 21$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

(*vi*) 4096 = $\sqrt[3]{4096} = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 = 16$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(*vii*) $8000 = \sqrt[3]{8000} = (4 \times 4 \times 4) \times (5 \times 5 \times 5)$

$= 4 \times 5 = 20$

4	8000
4	2000
4	500
5	125
5	25
5	5
	1

(*viii*) $3375 = \sqrt[3]{3375} = (5 \times 5 \times 5) \times (3 \times 3 \times 3)$

$= 5 \times 3 = 15$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

Percent and Percentage

[Chapter – 6]

1) What percent of 90 is 18 ?.

Ans. Let x% of 90 = 18, then

$90 \times \frac{x}{100} = 18$

$\Rightarrow \frac{9x}{10} = 18 \Rightarrow x = \left(18 \times \frac{10}{9}\right) = 20$

Hence, 20 % of 90 is 18.

2) 28 % of a number is 42. Find the number.

Ans. Let the required number be x.

Then, 28% of x = 42

$x = \frac{28}{100} = 42$

$x = \left(42 \times \frac{100}{28}\right) = 150$

Thus, the required number is 150.

3) The money spent on the repairs of a house was 2 % of its value. If the cost of repair is Rs. 9600. Find the cost of the house.

Ans. Let the cost of the house be Rs. x,

then 2 % of x = 9600

$\Rightarrow x \times \frac{2}{100} = 9600 \Rightarrow x = \times \left(9600 \times \frac{100}{2}\right) = 480000$

Thus, the cost of the house is Rs. 480000.

4) 1200 boys and 600 girls are examined in a test. 42 % of the boys and 30% of

the girls passed. Find the percentage of house who failed.

Ans. Total member of students = 1200 + 600 = 1800

No. of students who passed = (42 % of 1200) + (30 % of 600)

$$= \left(1200 \times \frac{42}{100}\right) + \left(600 \times \frac{30}{100}\right) = 504 + 180 = 684$$

No. of failure = 1800 – 1684 = 1116

$$\therefore \text{ Percentage of failures} = \left(\frac{1116}{1800} \times 100\right)\% = 62\%$$

5) Find the number which when increased by 12 % becomes 840.

Ans. Let the required number be x

$$\text{Increase} = 12\ \% \text{ of } x = \left(x + \frac{12}{100}\right) = \frac{3x}{25}$$

$$\text{Increase number} = \left(x + \frac{3x}{25}\right) = \frac{28x}{25}$$

$$\frac{28x}{25} = 840, \Rightarrow x = \left(840 \times \frac{25}{28}\right) = 750$$

Hence, the required number is 750.

6) By what number must a given number be multiplied to increase it by 20 %

Ans. Let the given number be x.

$$\text{Increase in its value} = 20\ \% \text{ of } x = \left(x \times \frac{20}{100}\right) = \frac{x}{5}$$

$$\text{Increase value} = \left(x + \frac{x}{5}\right) = \frac{6x}{5}$$

Hence, for an increase of 20 % , the given number should be multiplied by $\frac{6}{5}$

7) After deducting 6 % of a bill, the amount to be paid is Rs. 658. How much was the original bill ?

Ans. Let the amount of the original bill be Rs. x

$$\text{Deduction} = 6\ \% \text{ of } x = \text{Rs. } \frac{6}{100} \times 6 = \frac{3x}{50}$$

$$\text{Balance to be paid} = \text{Rs. } \left(x - \frac{3x}{50}\right) = \frac{47x}{50} = 658$$

$$x = 658 \times \frac{50}{47} = 700$$

Hence, the amount of the original bill is Rs. 700.

Profit & Loss, Discount & Tax

[Chapter – 7]

1) Raja bought a second hand scooter for Rs. 18000 and spent Rs. 1600 on its repair. Then he had to sell if for Rs. 17640. Find his loss percent.

Ans. C. P of the scooter = Rs. 18000

Money spent on its repair = Rs.1600

Net C.P. of the scooter = Rs. 18000 + Rs. 1600 = Rs.19600

S.P. of the scooter = Rs. 17640

Since CP > SP

Loss = CP – SP = 19600 – 17640 = Rs. 1960

Loss % = $\left(\frac{\text{Loss}}{\text{CP}} \times 100\right)\% = \left(\frac{1960}{19600} \times 100\right)\% = 10\%$

2) If the cost price of 8pens is equal to the selling price of 6 pens. Find the gain percent. .

Ans. Let C.P of the 8 pens = S.P. of 6 pens = Rs. x

Then, CP of each pen = Rs. $\frac{x}{8}$

SP of each pen = Rs. $\frac{x}{6}$

Now, $\frac{x}{6} > \frac{x}{8} \Rightarrow$ SP > CP

Gain = SP – CP = Rs. $\left(\frac{x}{6}-\frac{x}{8}\right)$ = Rs. $\left(\frac{4x-3x}{24}\right)$ = Rs. $\frac{x}{24}$

Gain % = $\left(\frac{\text{gain}}{\text{CP}} \times 100\right)\% = \left(\frac{x}{24} \times \frac{8}{x} \times 100\right)\% = \frac{100}{3}\%$

3) Kitu bought a bicycle for Rs. 6500. He had to sell it at a loss of 6% for how

much did he sell it ? .

Ans. C.P = Rs. 6500, Loss % = 6%

$$SP = \frac{100-l\%}{100} \times CP$$

$$= \left(\frac{100-6}{100} \times 6500\right) = \frac{94}{100} \times 6500 = Rs.110$$

Thus, Kitu sold the bicycle for Rs. 6110.

4) Mohesh sold a table for Rs. 1840 at a loss of 8 %. At what price did he purchase it ? .

Ans. SP = Rs. 1840, Loss = 8%

$$CP = \left\{\frac{100}{100-l\ \%} \times SP\right\} = \frac{100}{100-8} \times 1840 = Rs.\ \frac{100}{92} \times 1840 = Rs.2000$$

Hence, Mahesh purchased the table for Rs. 2000.

5) On selling a fan Rs. 1620, a shopkeeper loses 10%. For what amount should he sell it to gain 5 % ? .

Ans. Gain, SP = Rs. 1620, Loss = 10%

$$CP = Rs.\ \left(\frac{100}{100-l\%}\right) \times SP = Rs.\ \left(\frac{100}{100-10} \times 1620\right)$$

$$= Rs.\ \left(\frac{100}{90} \times 1620\right) = Rs.1800$$

Now, CP = Rs. 1800, Gain % = 5 %

$$SP = Rs.\ \left\{\frac{100+g\ \%}{100} \times CP\right\} = \frac{100+5}{100} \times 1800$$

$$= Rs.\ \frac{105}{100} \times 1800 = Rs.1890$$

Thus, SP = Rs. 1890.

6) A man had two bicycles for Rs. 6000 each, gaining 20% on the one and losing 20% on the other. Find this gain or loss percent on the whole transaction. .

Ans.

1st Bicycle	2nd Bicycle
SP = Rs. 6000, gain % = 20%	SP = Rs. 6000, loss % = 20%

$$CP = \frac{100}{100+g\%} \times SP$$

$$= Rs. \left\{\frac{100}{100+20} \times 6000\right\} = \frac{100}{120} \times 6000$$

= Rs. 5000

$$CP = \frac{100}{100+l\,\%} \times SP$$

$$= Rs. \left\{\frac{100}{100-20} \times 6000\right\} = \frac{100}{80} \times 6000$$

= Rs. 7500

Total CP of 2 bicycle = Rs. 5000 + Rs. 7500 = Rs. 12500

Total SP of 2 bicycles = Rs. 6000 x 2 = Rs. 12000

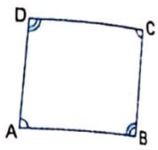
Loss = Rs. 12500 – Rs. 1200 = Rs. 500

$$\text{Loss } \% = \frac{L}{CP} \times 100 = \left(\frac{500}{12500} \times 100\right)\% = 4\%$$

Thus, the man losses 4% on the whole transaction.

Quadrilaterals

[Chapter – 17]



Example 1 : *If the opposite angles of a quadrilateral are equal, show that it is a parallelogram.*

Given : A quadrilateral ABCD in which ∠A = ∠C and ∠B = ∠D.

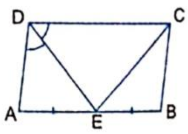
To prove : ABCD is a parallelogram.

Proof.

Statement	Reason
1. ∠A + ∠B + ∠C + ∠D = 360°	Sum of the ∠s of a quadrilateral is 360°.
2. 2∠A + 2 ∠B = 360°	∠C = ∠A and ∠D = ∠B (Given)
3. ∠A + ∠B = 180°	Halves of equals are equal.
4. AD ∥ BC	AB cuts AD and BC such that Co-Int. ∠s are supplementary.
5. AB ∥ DC	Similarly.
∴ ABCD is a parallelogram.	From **4** and **5**.

Example 2. *In the given figure, ABCD is a parallelogram; E is the mid-point of AB and DE bisects ∠D. Prove that :* (i) *BC = BE* (ii) *CE bisects ∠C* (iii) *∠DEC = 90°.*

Solution :



(i) To prove : BC = BE.

Proof.

Statement	Reason
1. ∠CDE = ∠ EDA	Given, DE bisects ∠D.
2. ∠CDE = ∠AED	Alt. Int. ∠s equal, as DC ∥ AB and DE is the transversal.
3. ∠ EDA = ∠AED	From **1** and **2**.
4. AD = AE	Sides opp. to equal ∠s are equal.
5. AD = BC	Opposite sides of a ∥gm are equal.
6. AE = BC	From **4** and **5**.
7. AE = BE	Given, E is the mid-point of AB.
8. BC = BE	From **6** and **7**.

(ii) To prove : CE bisects ∠C.

Proof

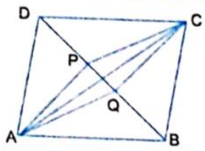
Statement	Reason
1. ∠ BCE = ∠ BEC	BC = BE and ∠s opposite to equal sides are equal.
2. ∠ DCE = ∠ BEC	Alt. Int. ∠s equal, as DC ∥ AB and CE is the transversal.
3. ∠ BCE = ∠ DCE	From **1** and **2**.
4. CE bisects ∠C	

(iii) To prove : ∠DEC = 90°

Proof.

Statement	Reason
1. ∠ADC + ∠BCD = 180°	Sum of Co-Int. ∠s is 180°, as AD ∥ BC and DC is the transversal.
2. $\frac{1}{2}$ ∠ADC + $\frac{1}{2}$ ∠BCD = 90°	Halves of equals are equal.
3. ∠CDE + ∠DCE = 90°	$\frac{1}{2}$ ∠ADC = ∠CDE and $\frac{1}{2}$ ∠BCD = ∠DCE.
4. ∠CDE + ∠DCE + ∠CED = 180°	Sum of the ∠s of a triangle is 180°
5. ∠CED = 90°	From **3** and **4**.

Example 3 : *In the given figure, ABCD is a parallelogram. The bisectors of ∠A and ∠C meet the diagonal BD at P and Q respectively. Show that PCQA is a parallelogram.*



Given : ABCD is a ||gm; AP bisects ∠A *and* CQ *bisects* ∠C.

To prove : PCQA is a ||gm.

Construction : Join AC.

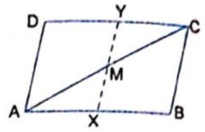
Proof.

Statement	Reason
1. ∠ BAP = $\frac{1}{2}$∠A	AP bisects ∠A.
2. ∠ DCQ = $\frac{1}{2}$∠C	CQ bisects ∠C.
3. ∠ BAP = ∠ DCQ	∠A = ∠C, opposite ∠s of a ||gm are equal ⇒ $\frac{1}{2}$∠A = $\frac{1}{2}$∠C.
4. ∠ BAC = ∠ DCA	Alt. Int. ∠s equal, since AB ∥ DC and CA is the transversal.
5. ∠ BAP − ∠ BAC = ∠ DCQ − ∠ DCA	From **3** and **4**.
6. ∠ PAC = ∠QCA	
7. AP ∥ QC	AP and CQ are cut by transversal AC forming Alt. Int. ∠s equal.
8. PC ∥ AQ	Similarly.
∴ PCQA is a parallelogram.	From **7** and **8**.

Example 4 : *In the given figure, ABCD is a parallelogram; M is the mid-point of AC; X and Y are points on AB and DC respectively, such that AX = CY.*

Prove that : (i) *ΔAXM ≅ ΔCYM.*

(ii) *XMY is a straight line.*



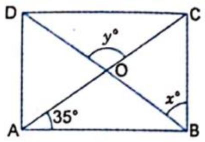
Given : ABCD is a ||gm, M is the mid-point of AC; X and Y are points on AB and CD such that AX = CY.

To prove : (i) ΔAXM ≅ ΔCYM. (ii) XMY is a straight line.

Proof.

Statement	Reason
1. In Δ AXM and Δ CYM, we have :	
(i) AX = CY	Given.
(ii) AM = MC	Given, M is the mid-point of AC.
(iii) ∠ XAM = ∠ MCY	Alt. Int. ∠*s* equal, as AB ∥ DC and CA is the transversal.
2. Δ AXM ≅ Δ CYM	SAS – axiom of congruence.
3. ∠AMX = ∠CMY	c.p.c.t.
4. ∠ XMC = ∠ XAM + ∠AXM	Ext. ∠ of Δ AMX = Sum of Int. opp. ∠*s*.
5. ∠CMY + ∠XMC = ∠XAM + ∠AXM + ∠AMX	From **3** and **4**.
6. ∠CMY + ∠XMC = 180°	Sum of the ∠*s* of a triangle is 180°.
7. XMY is a straight line.	Sum of adjacent ∠*s* is 180°.

Example 5 : *In the adjoining figure, ABCD is a rectangle. Find the values of x and y.*



Solution : We know that diagonals of a rectangle are equal and bisect each other.

So, AC = BD ⇒ 2AO = 2BO

⇒ AO = BO

⇒ ∠ABO = ∠OAB = 35° *[Angles opposite to equal sides are equal]*

Now, ∠ABO + ∠OBC = 90°

⇒ 35° + x° = 90° ⇒ x° = (90° – 35°) = 55°.

In ΔOAB, ∠OAB + ∠ABO + ∠AOB = 180°

⇒ 35° + 35° + ∠AOB = 180°

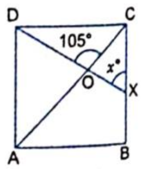
⇒ ∠AOB = 180° – (35° + 35°) = (180° – 70°) = 110°.

∠DOC = ∠AOB *[Vert. opp ∠s]*

⇒ y° = 110°

Hence, x = 55, y = 110.

Example 6 : *In the adjoining figure, ABCD is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that ∠ COD = 105° and ∠ OXC = x°. Find the value of x.*



Solution : The angles of a square are bisected by the diagonals.

∴ ∠OCX = 45° [∵ *∠DCB = 90° and CA bisects ∠DCB*]

Also, ∠COD + ∠COX = 180° *[Linear pair]*

⇒ 105° + ∠COX = 180° ⇒ ∠COX = (180° – 105°) = 75°.

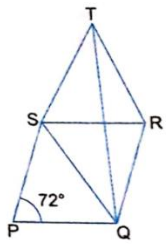
Now in ΔCOX, we have :

∠OCX + ∠COX + ∠OXC = 180° ⇒ 45° + 75° + ∠OXC = 180°

⇒ ∠OXC = (180° – 120°) = 60°.

Hence, x = 60.

Example 7 : *In the adjoining figure, PQRS is a rhombus and RST is an equilateral triangle. T and Q lie on opposite sides of RS. If ∠SPQ = 72°, calculate ∠RQT and ∠SQT.*



Solution : PQRS is a rhombus ⇒ ∠QRS = ∠SPQ = 72°.

RST is an equilateral Δ ⇒ ∠SRT = 60°.

∠QRT = ∠QRS + ∠SRT = 72° + 60° = 132°

Now, SR = RT *[RST is an equilateral Δ]*

and SR = QR *[PQRS is a rhombus]*

⇒ RT = QR

⇒ ∠RQT = ∠RTQ *[∠s opp. to equal sides of a Δ]*

But, ∠RQT + ∠QRT + ∠RTQ = 180° *[Sum of ∠s of a Δ]*

⇒ ∠RQT + ∠RTQ = (180° – 132°) = 48°

⇒ ∠RQT = $\frac{1}{2}$ × 48° = 24°. [∵ *∠RQT = ∠RTQ*]

In ΔSPQ, SP = PQ ⇒ ∠PSQ = ∠PQS ⇒ ∠PSQ = $\frac{1}{2}$(180° – 72°) = 54°.

But, ∠SQR = ∠PSQ = 54° *[Alt. int. ∠s, as PS ∥ QR]*

∴ ∠SQT = ∠SQR – ∠RQT = (54° – 24°) = 30°.

MODEL QUESTION BANK

SECOND TERM

CLASS – VIII

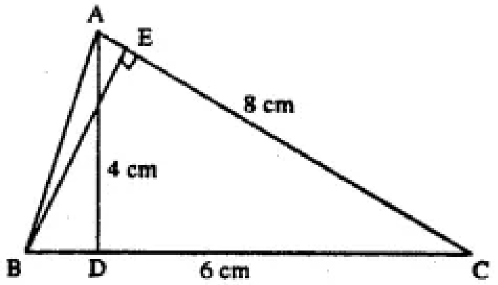
MATHEMATICS

New

Perimeter and area of plane figure

[Chapter – 23]

Question 1. Two sides of a triangle are 6 cm and 8 cm. If height of the triangle corresponding to 6 cm side is 4 cm ; :
(i) area of the triangle
(ii) height of the triangle corresponding to 8 cm side.
Solution:



$$BC = 6 \text{ cm}$$
$$\text{height } AD = 4 \text{ cm}$$
$$\text{area of } \Delta = \frac{1}{2} \text{base} \times \text{height}$$
$$= \frac{1}{2} \times BC \times AD$$
$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$
$$\text{Again area of } \Delta = \frac{1}{2} AC \times BE$$
$$12 = \frac{1}{2} \times 8 \times BE$$
$$\therefore \quad BE = \frac{12 \times 2}{8}$$
$$BE = 3 \text{ cm}$$
$\therefore$ (*i*) 12 cm^2 (*ii*) 3 cm Ans.

Question 2.

The sides of a triangle are 16 cm, 12 cm and 20 cm. Find :

(i) area of the
triangle ;
(ii) height of the triangle, corresponding to the largest side ;
(iii) height of the triangle, corresponding to the smallest side.

Solution: Sides of Δ are a = 20 cm, b = 12 cm c = 16 cm

$$S = \frac{a+b+c}{2}$$

$$= \frac{20+12+16}{2}$$

$$= \frac{48}{2} = 24$$

$$\text{area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-20)(24-12)(24-16)}$$

$$= \sqrt{24 \times 4 \times 12 \times 8}$$

$$= \sqrt{12 \times 2 \times 4 \times 12 \times 2 \times 4}$$

$$= \sqrt{12 \times 12 \times 4 \times 4 \times 2 \times 2}$$

$$= 12 \times 4 \times 2 = 96 \text{ cm}^2$$

AD is height of Δ corresponding to largest side.

$$\therefore \quad \frac{1}{2} \times BC \times AD = 96$$

$$\frac{1}{2} \times 20 \times AD = 96$$

$$AD = \frac{96 \times 2}{20}$$

$$AD = 9{\cdot}6 \text{ cm}$$

BE is height of Δ corresponding to smallest side.

$$\therefore \quad \frac{1}{2} AC \times BE = 96$$

$$\frac{1}{2} \times 12 \times BE = 96$$

$$BE = \frac{96 \times 2}{12}$$

$$BE = 16 \text{ cm}$$

(i) 96 cm^2 *(ii)* 9·6 cm *(iii)* 16 cm **Ans.**

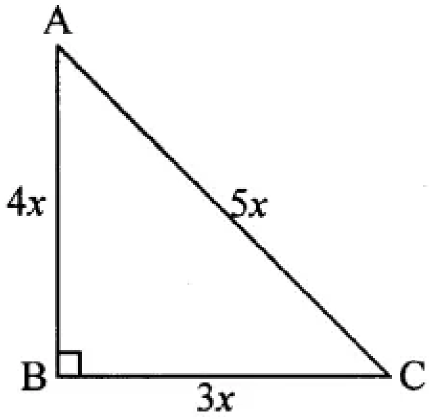
Question 3. The lengths of the sides of a triangle are in the ratio 4 : 5 : 3 and its perimeter is 96 cm. Find its area. Solution:

Let the sides of the triangle ABC be 4x, 5x and 3x

Let AB = 4x, AC = 5x and BC = 3x

Perimeter = 4x + 5x + 3x = 96

=> 12x = 96 $\Rightarrow \quad x = \frac{96}{12}$



$\therefore x = 8$

$\therefore$ Sides are

BC = 3(8) = 24 cm, AB = 4 (8) = 32 cm,

AC = 5(8) = 40 cm

Since $(AC)^2 = (AB)^2 + (BC)^2$

$[\because (5x)^2 = (3x)^2 + (4x)^2]$

$\therefore$ By Pythagorus Theorem, $\angle B = 90°$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ (BC) (AB) $= \frac{1}{2}$ (24) (32)

$= 12 \times 32 = 384 \text{ cm}^2$

Question 4.

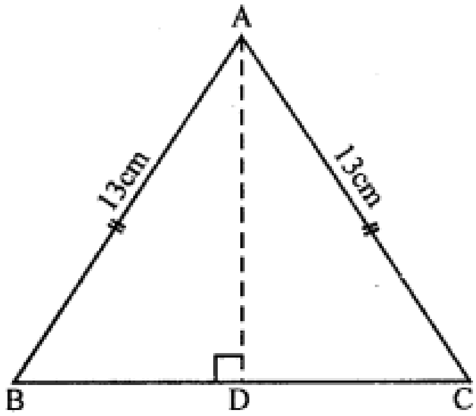
One of the equal sides of an isosceles triangle is 13 cm and its perimeter is 50 cm. Find the area of the triangle.

Solution:

In isosceles Δ ABC

AB = AC = 13 cm But perimeter = 50 cm

Solution:



$\therefore$ BC = 50 – (13 + 13) cm

= 50 – 26 = 24 cm

AD $\perp$ BC

$\therefore$ AD = DC = $\frac{24}{2}$ = 12 cm.

In right ΔABD,

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$(13)^2 = AD^2 + (12)^2$

$\Rightarrow 169 = AD^2 + 144$

$\Rightarrow AD^2 = 169 - 144$

$= 25 = (5)^2$

$\therefore$ AD = 5 cm.

Now area of ΔABC = $\frac{1}{2}$ Base × Altitude

$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 24 \times 5 = 60\ cm^2$

Question 5.

The area of a square is 169 cm^2. Find its:
(i) one side
(ii) perimeter

Solution:

Let each side of the square be x cm.

Its area = x^2 = 169 (given)
x = $\sqrt{169}$
x = 13 cm
(i) Thus, side of the square = 13 cm
(ii) Again perimeter = 4 (side) = 4 x 13 = 52 cm

Question 6.

The length of a rectangle is 16 cm and its perimeter is equal to the perimeter of a square with side 12.5 cm. Find the area of the rectangle.

Solution:

Length of the rectangle = 16 cm

Let its breadth be x cm

Perimeter = 2 (16 + x) = 32 + 2x

Also perimeter = 4(12.5) = 50 cm.

According to statement,

32 + 2x = 50

=> 2x = 50 – 32 = 18

=> x = 9

Breadth of the rectangle = 9 cm.

Area of the rectangle (l x b)=

16 x 9 = 144 cm

Question 7.

The perimeter of a square is numerically equal to its area. Find its area.

Solution:

Let each side of the square be x cm.

Its perimeter = 4x,

Area =x^2

By the given condition $4x = x^2$

=> $x^2 - 4x = 0$

=> $x (x - 4) = 0$

=> $x = 4$ [$x \neq 0$]

Area = $x^2 = (4)^2 = 4 \times 4 = 16$ sq.unit

Question 8. The two parallel sides and the distance between them are in the ratio 3 : 4 : 2. If the area of the trapezium is

175 cm^2; find its height.
Solution:

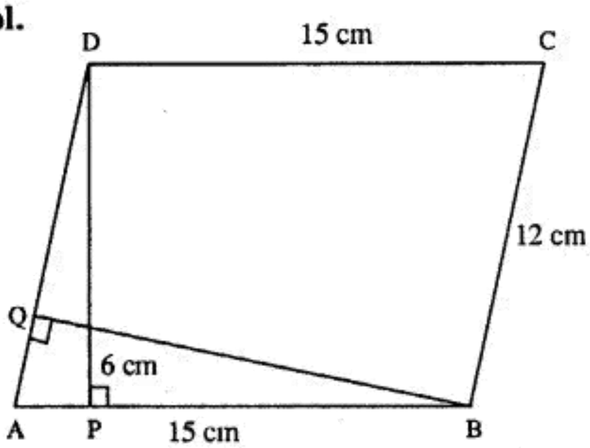
Let the two parallel sides and the distance between them be 3x, 4x, 2x cm respectively Area = (sum of parallel sides) x (distance between parallel sides)

= (3x + 4x) x 2x = 175 (given)
=> 7x x x = 175
2
=> 7x = 175
2
=> x = 25
=> x = 5
Height i.e. distance between parallel sides = 2x = 2 x 5 = 10 cm

Question 9.

A parallelogram has sides of 15 cm and 12 cm; if the distance between the 15 cm sides is 6 cm; find the distance between 12 cm sides.
Solution:

Sol.



Base, AB = 15 cm

Distance between 15 cm sides

i.e. height DP = 6 cm

$\therefore$ Area of ||gm = Base $\times$ height

= AB $\times$ DP

= $15 \times 6 = 90$ cm^2

Let BQ be distance between 12 cm sides

$\therefore$ AD $\times$ BQ = area of ||gm ABCD

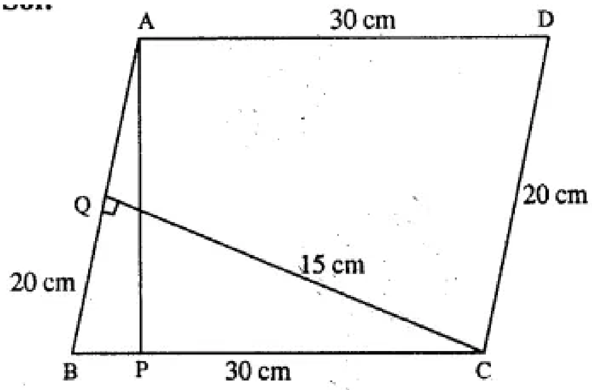
$\therefore$ $12 \times$ BQ = 90

$$BQ = \frac{90}{12}$$

BQ = 7.5 cm

Question 10.

A parallelogram has sides of 20 cm and 30 cm. If the distance between its shorter sides is 15 cm; find the distance between the longer sides.



Let ABCD be the ||gm in which BC = 30 cm and CD = 20 cm

Distance between shorter sides,

i.e. CQ = 15 cm

∴ area of ||gm = AB×CQ = 20×15 = 300 cm^2

Again BC×AP = Area of || gm

30×AP = 300

$AP = \frac{300}{30}$

AP = 10 cm

∴ Distance between larger sides is = 10 cm **Ans.**

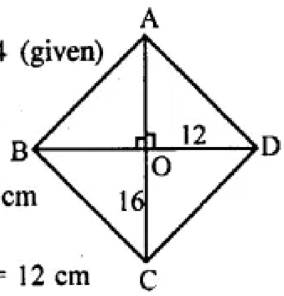
Question 11.

The length of the diagonals of a rhombus is in the ratio 4 : 3. If its area is 384 cm^2, find its side. Solution:

Let the lengths of the diagonals of rhombus are 4x, 3x.

∴ Area of the rhombus = $\frac{1}{2}$ (Product of its diagonals)

$= \frac{1}{2}(4x \times 3x) = 384$ (given)



$\Rightarrow 6x^2 = 384 \Rightarrow x^2 = 64$

$\Rightarrow x = 8$ cm

∴ Diagonals are 4×8=32cm and 3(8) = 24 cm.

∴ OC = 16 cm and OD = 12 cm

∴ Side DC = $\sqrt{OC^2 + OD^2}$

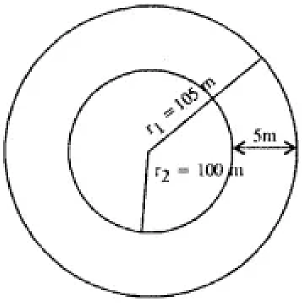
∴ Side DC = $\sqrt{16^2 + 12^2}$

[By Pythagoras Theorem in ΔDOC]

$= \sqrt{256+144} = \sqrt{400} = 20$ cm

Hence, side of the rhombus = 20 cm.

Question 12.
A circular field of radius 105m has a circular path of uniform width of 5m along and inside its boundary. Find the area of the path.



Radius of circular field, $r_1 = 105$ m

Width of path $= 50$ m

$\therefore$ Radius of inner circle, $r_2 = 105 - 5 = 100$m

$\therefore$ Area of path $= \pi r_1^2 - \pi r_2^2$

$$= \frac{22}{7}[(105)^2 - (100)^2]$$

$$= \frac{22}{7}(105 + 100)$$

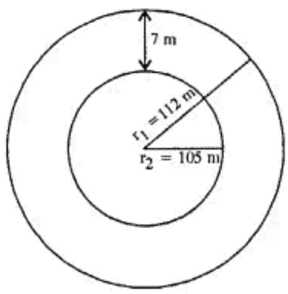
$$(105 - 100)$$

$$= \frac{22}{7} \times 205 \times 5$$

$$= \frac{22550}{7} \text{ m}^2$$

$$= 3221\frac{3}{7} \text{ m}^2 \textbf{ Ans.}$$

Question 13. There is a path of uniform width 7 m round and outside a circular garden of diameter 210 m. Find the area of the path.



Diameter $= 210$ m

Radius of inner circule $r_2 = 105$ m

Width $= 7$m

Radius of outer circle $r_1 = 105 + 7 = 112$ m

$\therefore$ Area of path $= \pi r_1^2 - \pi r_2^2$

$= \pi[r_1^2 - r_2^2]$

$$= \frac{22}{7}(r_1 + r_2)(r_1 + r_2)$$

$$= \frac{22}{7}(112 + 105)$$

$$(112 - 105)$$

$$= \frac{22}{7} \times 217 \times 7$$

$$= 4774 \text{ m}^2 \textbf{ Ans.}$$

Question 14.
A wire, when bent in the form of a square encloses an area of 484 cm^2. Find :
(i) one side of the square ;
(ii) length of the wire ;
(iii) the largest area enclosed; if the same wire is bent to form a circle.
Solution:
(i) Area of Square = 484 cm^2
Side of Square = $\sqrt{\text{Area}} = \sqrt{484}$ = 22 cm
(ii) Perimeter, i.e. length of wire = 4 x 22 = 88 cm
(iii) Circumference of circle = 88 cm =>

$$2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7}{2 \times 22}$$

$$r = 14 \text{ cm}$$

$$\therefore \quad \text{The largest area enclosed} = \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ cm}^2$$

Hence (*i*) 22 cm (*ii*) 88 cm (*iii*) 616 cm^2 **Ans.**

SURFACE AREA, VOLUME AND CAPACITY

[Chapter – 24]

Question 1.

The length, breadth and height of a cuboid are in the ratio 5 : 3 : 2. If its volume is 240 cm^3; Find its dimensions.(Dimensions means: its length, breadth and height). Also find the total surface area the cuboid.
Solution:
Let length of the given cuboid = 5x
Breadth of the given cuboid = 3x
Height of the given cuboid = 2x
Volume of the given cuboid = Length x Breadth x Height
= x 3x x 2x = 30 x^3
= But we are given volume = 240cm^3
= 30x^3 = 240 cm^3
= x =
= x = 8
x = 2 cm
Length of the given cube = 5x = 5 x 2 = 10 cm
Breadth of the given cube = 3x = 3 x 2 = 6 cm
Height of the given cube = 2x = 2 x 2 = 4cm
Total surface area of the given cuboid = 2(l x b + b x h + h x l)
= 2(10 x 6 + 6 x 4 + 4 x 10) = 2(60 + 24 + 40) = 2 x 124 = 248 cm^2

Question 2.

The length, breadth and height of a cuboid are in the ratio 6 : 5 : 3. If its total surface area is 504 cm^2; dimensions. Also find the volume of the cuboid.
Solution:

Let length of the cuboid = 6x
Breadth of the cuboid = 5x
Height of the cuboid = 3x
Total surface area of the given cuboid = 2 (l x b + b x h + h x l)
= 2(6x x 5x + 5x x 3x + 3x x 6x) = 2(30×2 + 15×2 + 18×2)
= 2 x 63×2 = $126x^2$
= But we are given total surface area of the given cuboid = 504 cm
= $126x^2$ = 504 cm^2
=> x = 504 / 126
= => x = 4
Length = 66x = 66 x 4=24cm, breadth=5x=5X4=20cm, Height=3x=3 X 4=12
Volume of the cuboid = lbh = 24X20X12=24X240=5760$\mathbf{cm}^3$

Question 3. Find the volume and total surface area of a cube whose each edge is :
(i) 8 cm (ii) 2 m 40 cm.
Solution:
(i) Edge of the given cube = 8 cm

Volume of the given cube = $(Edge)^3$ = $(8)^3$ = 8 x 8 x 8 = 512 cm

Total surface area of a cube = $6(Edge)^2$ = 6 x $(8)^2$ = 384 cm
(ii) Edge of the given cube = 2 m 40 cm = 2.40 m

Volume of a cube = (Edge)

= $(2.40)^3$ = 2.40 x 2.40 x 2.40 = 13.824 m^3

Volume of the given cube = $(\mathbf{2.40})^3$= 2.40 x 2.40 x 2.40 = 13.824 m^2

Total surface area of the given cube = 6 x 2.4 x 2.4 = 34.56 m^2

Question 4. The total surface area of a cube is 216 cm2. Find its volume.
Solution:
$6(Edge)^2$ = Total surface area of a cube
$6(Edge)^2$ = 216 cm^2
=> $(Edge)^2$ =
=> $(Edge)^2$ = 36
=> Edge = √ 36
=> Edge = 6 cm
Volume of the given cube = $(Edge)^3$ = $(6)^3$ = 6 x 6 x 6 = 216 cm^3

Question 5.
A solid cuboid of metal has dimensions 24 cm, 18 cm and 4 cm. Find its volume.
Solution:

Length of the cuboid = 24 cm
Breadth of the cuboid = 18 cm
Height of the cuboid = 4 cm

Volume of the cuboid = l x b x h = 24 x 18 x 4 = 1728 cm^3

Question 6.
Find the volume of wood required to make a closed box of external dimensions 80 cm, 75 cm and 60 cm, the thickness of walls of the box being 2 cm throughout.
Solution: External length of the closed box
= 80 cm
External Breadth of the closed box = 75 cm
External Height of the closed box = 60 cm

External volume of the closed box = 80 x 75 x 60 = 360000 cm

Internal length of the closed box = 80 – 4 = 76 cm
Internal Breadth of the closed box = 75 – 4 = 71 cm
Internal Height of the closed box = 60 – 4=56 cm
Internal volume of the closed box = 76 x 71 x 56 cm = 302176 cm
Volume of the wood = 360000 – 302176 = 57824cm^3

Question 7.

Three solid cubes of edges 6 cm, 10 cm and x cm are melted to form a single cube of edge 12 cm, value of x.
Solution:
Edge of first cube = 6 cm^3
Volume = $(6)^3$ = 216 cm^3
Edge of second cube = 10 cm
Volume = $(10)^3$ = 1000 cm^3
Edge of third cube = x
Volume = X^3

Edge of resulting cube = 12 cm

Volume = $(12)^3$ = 1728 cm^3
216 + 1000 + x^3 = 1728
X^3 = 1728 – 216 – 1000 = 512 = $(8)^3$
x = 8
Edge of third cube = 8 cm

Question 8.
The length of the diagonals of a cube is 8 √3 cm.
Find its:
(i) edge
(ii) total surface area

(iii) Volume

Solution:

(i) Length of diagonal of a cube = 8 √3 cm (ii) Total surface area= $6a^2$ = 6X64=384cm^2

Length of edge = = 8 cm

(iii) Volume=a^3 = $(8)^3$ $= 512cm^3$

INTEREST
[CHAPTER-8]

Question 1.

In what time will the interest on a certain sum of money at 6% be 5/8 of itself ?

Solution: Let P = Rs.8

$$\text{Interest} = \text{Rs.}8 \times \frac{5}{8} = \text{Rs.}5$$

$$R = 6\%$$

$$T = \frac{100 \times I}{P \times R}$$

$$= \frac{100 \times 5}{8 \times 6}$$

$$= \frac{500}{48} = \frac{125}{12} \text{ years}$$

$$= 10\frac{5}{12} \text{ years}$$

$$= 10 \text{ years } 5 \text{ months}$$

$$\left[\because \frac{5}{12} \text{ year} = \frac{5}{12} \times 12 \text{ months} = 5 \text{ months}\right]$$

$\therefore$ Time = 10 years 5 months

Question 2. Ashok lent out Rs.7000 at 6% and Rs.9500 at 5%. Find his total income from the interest in 3 years. Solution

In I case :

$$P = \text{Rs.}7000$$

$$R = 6\%$$

$$T = 3 \text{ years}$$

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$= \text{Rs.}\frac{7000 \times 6 \times 3}{100}$$

$$= \text{Rs.}1260$$

In II case :

$$P = \text{Rs.}9500$$

$$R = 5\%$$

$$T = 3 \text{ years}$$

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$= \text{Rs.}\frac{9500 \times 5 \times 3}{100}$$

$$= \text{Rs.}1425$$

Total income from the interest

$$= \text{Rs.}1260 + \text{Rs.}1425$$

$$= \text{Rs.}2685$$

Question 3. Raj borrows Rs.8,000; out of which Rs. 4500 at 5% and remainder at 6%. Find the total interest paid by him in 4 years.

Total sum borrowed by Raj = Rs.8000

In the First Case :

P = Rs.4500

R = 5%

T = 4 years

$$S.I. = \frac{P \times R \times T}{100}$$

$$= Rs.\frac{4500 \times 5 \times 4}{100}$$

= Rs.900

In the Second Case :

P = Rs.8000 - Rs.4500

= Rs.3500

R = 6%

T = 4 years

$$S.I. = \frac{P \times R \times T}{100}$$

$$= Rs.\frac{3500 \times 6 \times 4}{100}$$

$= 35 \times 6 \times 4 =$ Rs.840

Total interest paid by Raj

= Rs.900 + Rs.840

= Rs.1740

Question 4. A certain sum amounts to Rs.3825 in 4 years and to Rs.4050 in 6 years. Find the rate percent and the sum.

In 6 years sum amounts to = Rs.4050

In 4 years sum amounts to = Rs.3825

∴ Interest of 2 years = Rs.4050 - Rs.3825

= Rs.225

$$\text{Interest of 4 years} = Rs.\frac{225}{2} \times 4$$

= Rs.450

(∵ Rs.225 is interest for 2 years)

Now P = A—I

= Rs.3825 - Rs.450

= Rs.3375

I = Rs.450

T = 4 years

$$R = \frac{100 \times I}{P \times T}$$

$$= \frac{100 \times 450}{3375 \times 4}$$

$$= \frac{45000}{13500}\% = \frac{450}{135}\%$$

$$= \frac{10}{3}\% = 3\frac{1}{3}\%$$

∴ $R = 3\frac{1}{3}\%$

P = Rs.3375

Question 5. At what rate percent of simple interest will the interest on Rs.3750 be one-fifth of itself in 4years? To what will it amount in 15 years ?

Solution:

$$R = \frac{100 \times I}{P \times T}$$

$$= \frac{100 \times 750}{3750 \times 4}$$

$$= \frac{100 \times 750}{3750 \times 4}$$

$$= 5\%$$

Again, $P = \text{Rs.}3750$

Interest of 4 years $= \text{Rs.}750$

Interest of 1 year $= \text{Rs.}\frac{750}{4}$

Interest of 15 years $= \text{Rs.}\frac{750}{4} \times 15$

$$= \text{Rs.}\frac{750 \times 15}{4}$$

$$= \text{Rs.}\frac{5625}{2}$$

$$= \text{Rs.}2812{\cdot}50$$

Amount in 15 years will be

$$= \text{Rs.}3750 + \text{Rs.}2812{\cdot}50$$

$$= \text{Rs.}6562{\cdot}50$$

$\therefore$ Rate $= 5\%$

Amount in 15 years will be

$$= \text{Rs.}6562{\cdot}50$$

Question6. Mohan borrowed Rs. 15,000 for 3 years at 6% for the first year and 8% per annum for the second year and 10% for last year, compound interest. Calculate the amount that Mohan will pay at the end of 3 years.

Solution:

$= \text{Rs. } 15{,}000 + \text{Rs. } 900 = \text{Rs. } 15900$

For 2nd year

$P = \text{Rs. } 15900,\ R = 8\%,\ T = 1$ year

$\therefore$ Interest $= \frac{15{,}900 \times 8 \times 1}{100} = 159 \times 8 = \text{Rs. } 1272$

$\therefore$ Amount at the end of 2nd year

$= \text{Rs. } (15900 + 1272) = \text{Rs. } 17172$

For 3rd year

$P = \text{Rs. } 17172,\ R = 10\%,\ T = 1$ year

$\therefore$ Interest $= \frac{17172 \times 10 \times 1}{100} = \text{Rs. } 1717.20$

$\therefore$ Amount at the end of 3rd year

$= \text{Rs. } (17172 + 1717.20) = \text{Rs. } 18889.20$

$\therefore$ Compound interest $= 18889.20 - 15{,}000$

$= \text{Rs. } 3889.20$

Question 7. Rekha borrowed Rs. 40,000 for 3 years at 10% per annum compound interest. Calculate the interest paid by her for the second year.

Solution:

For 1st year

Principal = Rs. 40,000, Rate = 10%, Time = 1 year

$\therefore$ Interest = $\frac{40,000 \times 10 \times 1}{100}$ = 400 × 10 = Rs. 4000

$\therefore$ Amount at the end of 1st year = Rs. (40,000 + 4000) = Rs. 44,000

For 2nd year

P = Rs. 44,000, R = 10% ,T = 1 year

$\therefore$ Interest = Rs. $\frac{44,000 \times 10 \times 1}{100}$ = 440 × 10 = Rs. 4400

Thus interest earned in the second year = Rs. 4400

DIRECT AND INVERSE VARIATIONS [CHAPTER-10]

Question 1.

For 100 km, a taxi charges Rs. 1,800. How much will it charge for a journey of 120 km?

Let a charges of car is ₹ x in 120 km

Distance in (km)	1800	x
Taxi charges (₹)	100	120

Since it is the case of direct variation

$$\Rightarrow \frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{1800}{100} \times \frac{x}{120}$$

$$\Rightarrow 100x = 1800 \times 120$$

$$x = \frac{1800 \times 120}{100} = ₹2160$$

Question 2.

If 27 identical articles cost Rs. 1,890, how many articles can be bought for Rs.1,750?

Solution:

Let x number of articles be purchased in ₹1750

Cost (₹)	1890	1750
No. of articles	27	x

Since, it is a case of direct variation

$$\Rightarrow \frac{1890}{27} = \frac{1750}{x}$$

$$\Rightarrow x = \frac{1750 \times 27}{1890}$$

$= 25$ articles

Question 3.

7 kg of rice costs Rs. 1,120. How much rice can be bought for Rs. 3,680?

Rice : Cost :: Rice : Cost

7 kg : ₹1120 :: x kg : ₹3680

$$\therefore x = \frac{7 \times 3680}{1120} = 23 \text{ kg}$$

Question 4.

In a fort 150 men had provisions for 45 days. After 10 days, 25 men left the fort. How long would the food last at the same rate?

Solution:

After 10 days :

For 150 men, provision will last (45 – 10) days = 35 days

$\Rightarrow$ For 1 man, the provisions will last = 150 × 35 days

And for (150 – 25) = 125 men, the provisions will last for $= \frac{150 \times 35}{125} = 42$ days

Question 5.

72 men do a piece of work in 25 days. In how many days will 30 men do the same work?

Solution:

Men : Days :: Men : Days

72 : 25 :: 30 : x

$\therefore$ By inverse proportion

$72 \times 25 = 30 \times x$

$$\Rightarrow x = \frac{25 \times 72}{30}$$

$= 60$ days

Question 6.

If 56 workers can build a wall in 180 hours, how many workers will be required to do the same work in 70 hours?

Workers : Hours :: Workers : Hours

56 : 180 :: x : 70

$\therefore$ By inverse proportion

$56 \times 180 = x \times 70$

$$\Rightarrow x = \frac{180 \times 56}{70}$$

= 144 workers

Question 7.

50 labourers can dig a pond in 16 days. How many labourers will be required to dig an another pond, double in size in 20 days ?

Solution:

In 16 days for diging a pond labour reqd. = 50

In 1 day, labour reqd. = 50×16

In 20 days, labour reqd. = $\frac{50 \times 16}{20}$

In 20 days, with double work, labour reqd.

$$= \frac{50 \times 16 \times 2}{20} = 5 \times 8 \times 2 = 80$$

Hence labourers required = 80

Question 8.

If 12 men or 18 women can complete a piece of work in 7 days, in how many days can 4 men and 8 women complete the same work?

Solution :

12 men = 18 women

$$\therefore 4 \text{ m} = \frac{18}{12} \times 4$$

= 6 women

Total women in second case = 4 men + 8 women

6 + 8 = 14 women

18 women can do a piece of work in 7 days

Let 14 women will do the same work in x days

$\therefore$ 18 : 14 : : 7 : x

(Less women, more days)

$\Rightarrow$ 18 : 14 : : x : 7

(Using inverse proportion)

$$x = \frac{18 \times 7}{14}$$

= 9

$\therefore$ They will complete the work in 9 days

Question 9. If 3 men or 6 boys can finish a work in 20days, how long will 4 men and 12 boys take to finish the same work?

Solution:

3 men = 6 boys

4 men = x 4 = 8

Total boys in second case :

= 4 men + 12 boys = 8 + 12 = 20 boys

6 boys can do a piece of work in 20 days

$\therefore$ 6 : 20 : : 20 ; x

(More boys , less days)

$\Rightarrow$ 6 : 20 : : x : 20

(By inverse proportion)

$$x = \frac{20 \times 6}{20}$$

= 6 days

$\therefore$ They will do the work in 6 days

Question 10.

A family of 5 persons can be maintained for 20 days with Rs.2,480. Find, how long Rs.6944 maintain a family of 8 persons?

Solution:

A family of 5 persons can be maintained with

$$\text{Re.1 for} = \frac{20}{2480} \text{ days}$$

A family of 5 persons can be maintained with

$$\text{Rs.6944} = \frac{20}{2480} \times 6944 = \frac{1}{124} \times 6944$$

= 56 days

A family of 1 person can be maintained

= 56 × 5 days

A family of 8 persons can be maintained

$$= \frac{56 \times 5}{8} \text{ days}$$

= 7 × 5 days

= 35 days

TIME AND WORK
[CHAPTER-11]

Question 1. If 10 horses consume 18 bushels in 36 days, how long will 24 bushels last for 30 horses?

10 horses consume 18 bushels in

= 36 days

1 horse consumes 18 bushels in = 36 × 10 days

= 360 days

$$\text{30 horses consume 18 bushels in} = \frac{360}{30} \text{ days}$$

= 12 days

$$\text{30 horses consume 1 bushel in} = \frac{12}{18} \text{ days}$$

30 horses consume 24 bushels in

$$= \frac{12}{18} \times 24 \text{ days}$$

$$= \frac{2}{3} \times 24 \text{ days}$$

= 16 days

Question 2. A can do a piece of work in 20 days and B in 15 days. They worked together on it for 6 days and then A left. How long will B take to finish the remaining work?

A can do a piece of work in = 20 days

B can do a piece of work in = 15 days

$\therefore$ A's 1 day work $= \frac{1}{20}$

B's 1 day work $= \frac{1}{15}$

(A+B)'s 1 day work $= \frac{1}{20} + \frac{1}{15}$

$= \frac{3+4}{60} = \frac{7}{60}$

(A+B)'s 6 days work $= \frac{7}{60} \times 6 = \frac{7}{10}$

Remaining Work $= 1 - \frac{7}{10}$

$= \frac{10-7}{10} = \frac{3}{10}$

B can do 1 work in = 15 days

B can do $\frac{3}{10}$ work in $= 15 \times \frac{3}{10}$ days

$= \frac{45}{10}$ days $= \frac{9}{2}$ days $= 4\frac{1}{2}$ days

Question 3. A can finish a piece of work in 15 days and B can do it in 10days. They worked together for 2days and then B goes away. In how many days will A finish the remaining work.

A can finish a piece of work in = 15 days

B can finish a piece of work in = 10 days

$\therefore$ A's 1 day work $= \frac{1}{15}$

B's 1 day work $= \frac{1}{10}$

(A+B)'s 1 day work $= \frac{1}{15} + \frac{1}{10}$

$= \frac{2+3}{30} = \frac{5}{30} = \frac{1}{6}$

(A+B)'s 2 days work $= \frac{1}{6} \times 2 = \frac{1}{3}$

Remaining work which will be done by A alone

$= 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

$\therefore$ A can finish 1 work in = 15 days

A can finish $\frac{2}{3}$ work in $= 15 \times \frac{2}{3}$ days

$= \frac{30}{3}$ days $= 10$ days

Question 4. A can do a piece of work in 10 days ; B in 18 days; and A, B and C together in 4 days. In what time would C alone do it ?

Solution:

A can do a piece of work in = 10 days

B can do a piece of work in = 18 days

(A+B+C) can do a piece of work in = 4 days

$\therefore$ A's 1 day work $= \frac{1}{10}$

B's 1 day work $= \frac{1}{18}$

(A+B)'s 1 day work $= \frac{1}{10} + \frac{1}{18} = \frac{9+5}{90}$

$= \frac{14}{90} = \frac{7}{45}$

(A+B+C)'s 1 day work $= \frac{1}{4}$

$\therefore$ C's 1 day work $= \frac{1}{4} - \frac{7}{45}$

$= \frac{45-28}{180} = \frac{17}{180}$

$\therefore$ C can do the piece of work in $= \frac{180}{17}$ days

$= 10\frac{10}{17}$ days

PLAYING WITH NUMBERS
[CHAPTER-4]

Question 1. Find the quotient when 94 – 49 is divided by

(i) 9

(ii) 5

Difference of 94 and 49 is to be divided by

(*i*) 9 (*ii*) 5

Let $ab = 94$ and $ba = 49$

$\therefore$ $a = 9$ and $b = 4$

(*i*) The quotient of 94 – 49 *i.e.* $(ab - ba)$ when divided by 9 is $(a - b)$ *i.e.* $9 - 4 = 5$

$$\left(\because \frac{ab-ba}{9} = a-b\right)$$

(*ii*) The quotient of 94 – 49 *i.e.* $(ab - ba)$ when divided by 5 *i.e.* $(a - b)$ is 9

$$\left(\because \frac{ab-ba}{a-b} = 9\right)$$

Question 2.

Show that 527 + 752 + 275 is exactly divisible by 14.

Solution:

Property :

abc = 100a + 106 + c ....

(i) bca = 1006 + 10c + a ....

(ii) and cab = 100c + 10a + b ....

(iii)Adding,

(i), (ii) and (iii), we get abc + bca + cab = 111a + 111b + 111c = 111(a + b + c) = 3 x 37(a + b + c)

Now, let us try this method on

527 + 752 + 275 to check is it exactly divisible by 14

Here, a = 5, 6 = 2, c = 7

527 + 752 + 275 = 3 x 37(5 + 2 + 7) = 3 x 37 x 14

Hence, it shows that 527 + 752 + 275 is exactly divisible by 14

Question 3. If a = 6, show that

abc = bac.

Solution:

Given : a = 6

To show :abc = bac

Proof: abc = 100a + 106 + c ....(i)

(By using property 3)

bac = 1006 + 10a + c —(ii)

(By using property 3)

Since, a = 6

Substitute the value of a = 6 in equation (i) and (ii), we get

abc = 1006 + 106 + c ....(iii)

bac = 1006 + 106 + c ....(iv)

Subtracting (iv) from (iii) abc– bac = 0

abc = bac

Hence proved.

Question 4.

$$\begin{array}{r} A\,B \\ \times\ 6 \\ \hline \underline{B\,B\,B} \end{array}$$

Solution:

As we need B at unit place and B at ten's place,

∴ B = 4 as 6 × 4 = 24

Now we want to find A, 6 × A + 2 = 4 (at unit's place)

∴ A = 7

$$\begin{array}{r} 7\,4 \\ \times\ 6 \\ \hline \underline{4\,4\,4} \end{array}$$

Question 5.

$$\begin{array}{r} A\,B \\ \times\ 3 \\ \hline \underline{C\,A\,B} \end{array}$$

Solution:

As we need B at unit place and A at ten's place,

∴ B = 0 as $3 \times 0 = 0$

Now we want to find A, $3 \times A = A$ (at unit's place)

∴ A = 5, as $3 \times 5 = 15$

∴ C = 1

$$\begin{array}{r} 50 \\ \times 3 \\ \hline 150 \\ \hline \end{array}$$

Question6.

Find which of the following numbers are divisible by 3:

(i) 261 (ii) 111 (iii) 6657 (iv) 2574

Solution:

A number is divisible by 3 if the sum of its digits is divisible by 3,

So, all the numbers above are divisible by 3.

Question 7.

Find which of the following numbers are divisible by 4:

(i) 360 (ii) 3180 (iii) 5348 (iv) 7756

Solution:

A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

So, all the numbers above are divisible by 4.

Question 8.

Which of the following numbers are divisible by 11 :

(i) 2563

(ii) 8307

(iii) 95635

Solution:

A number is divisible by 11 if the difference of the sum of digits at the odd places and sum of the digits at even places is zero or divisible by 11.

So, 2563 is divisible by 11.

SETS

[CHAPTER- 5]

Question 1.

List the elements of the following sets :

(i) $\{x : x^2 - 2x - 3 = 0\}$

(ii) $\{x : x = 2y + 5; y \in N \text{ and } 2 \leq y < 6\}$

(iii) {x : x is a factor of 24}

(iv) $\{x : x \in Z \text{ and } x \leq 4\}$

(v) $\{x : 3x - 2 \leq 10, x \in N\}$

(vi) $\{x : 4 - 2x > -6, x \in Z\}$

Solution:

(i) $\{x : x^2 - 2x - 3 = 0\}$

$$x^2 - 2x - 3 = 0$$

$$\Rightarrow x^2 - 3x + x - 3 = 0$$

$$\Rightarrow x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (x+1)(x-3) = 0$$

$\therefore$ Either $x - 3 = 0$ Or $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

$\therefore$ Elements of the set $\{x : x^2 - 2x - 3 = 0\}$ are 3 and -1

(ii) $\{x : x = 2y + 5; y \in N \text{ and } 2 \leq y < 6\}$

$x = 2y + 5$

When $y = 2$, $\quad x = 2\times2+5 = 4+5 = 9$

When $y = 3$, $\quad x = 2\times3+5 = 6+5 = 11$

When $y = 4$ $\quad x = 2\times4+5 = 8+5 = 13$

When $y = 5$, $\quad x = 2\times5+5 = 10+5 = 15$

$\therefore$ Elements of the given set $\{x : x = 2y+5; y \in N \text{ and } 2 \leq y < 6\}$ are 9, 11, 13, 15

(iii) {x : x is a factor of 24}

$24 = 1\times24$

$24 = 2\times12$

$24 = 3\times8$

$24 = 4\times6$

$\therefore$ Elements of the given set {x : x is a factor of 24} are 1, 2, 3, 4, 6, 8, 12, 24

(iv) $\{x : x \in Z \text{ and } x^2 \leq 4\}$

When $x^2 = 4$

$$x = \pm\sqrt{4} = \pm2$$

When $x^2 = 1$

$$x = \pm\sqrt{1} = \pm1$$

When $x^2 = 0$

$$x = \sqrt{0} = 0$$

$\therefore$ Elements of the given set $\{x : x \in Z \text{ and } x^2 \leq 4\}$ are $+2, -2, +1, -1, 0$ or are $-2, -1, 0, 1, 2$

(v) $\{x : 3x-2 \leq 10, x \in N\}$

$$3x-2 \leq 10$$

$\Rightarrow$ $3x \leq 10+2$

$\Rightarrow$ $3x \leq 12$

$\Rightarrow$ $x \leq \frac{12}{3}$

$\leq$ $x \leq 4$

$\therefore$ Elements of the given set $\{x : 3x-2 \leq 10, x \in N\}$ are 1,2,3 and 4

(vi) $\{x : 4-2x > -6, x \in Z\}$

$$4-2x > -6$$

$$-4+4-2x > -6-4$$

(Subtracting 4 from both sides)

$$-2x > -10$$

$$-2x+2x+10 > -10+2x+10$$

[Adding $2x+10$ to both sides]

$$+10 > 2x$$

$$\frac{10}{2} > x$$

$$5 > x$$

$\therefore$ Elements of the given set $\{x : 4-2x>-6, x \in Z\}$ are 4,3,2,1,0,−1........

Question 2. State whether each of the following sets is a finite set or an infinite set:

(i) The set of multiples of 8.

(ii) The set of integers less than 10.

(iii) The set of whole numbers less than 12.

(iv) $\{x : x = 3n - 2, n \in W, n \leq 8\}$

(v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

(vi) $\{x : x = \frac{n-2}{n+1}, n \in w\}$

Solution

(i) The set of multiples of 8

= {8, 16, 24, 32, ...}

It is an infinite set.

(ii) The set of integers less than 10

= {9, 8, 7, 6, 5, 4, 3, 2, 1, −1, −2, ...}

It is an infinite set.

(iii) The set of whole numbers less than 12

= {11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0}

It is a finite set.

(iv) $\{x : x = 3n - 2, n \in W, n \leq 8\}$

Substituting the value of n = (0, 1, 2, 3, 4, 5. 6, 7 and 8) we get

= {−2, 1, 4, 7, 10, 13, 16, 19, 22}

It is finite set.

(v) $\{x : x = 3n - 2, n \in Z, n \leq 8\}$

= {22, 19, 16, 13, 10, 7, 4, 1, −2, −5, ... }

It is infinite set.

(vi) $\{x : x = \frac{n-2}{n+1}, n \in W\}$

$\{-2, -\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5} ....\}$

It is infinite set.

Question 3.

Answer, whether the following statements are true or false. Give reasons.

(i) The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are equivalent sets.

(ii) If E = {factors of 16} and F = {factors of 20}, then E = F.

(iii) The set A = {integers less than 20} is a finite set

(iv) If A = {x : x is an even prime number}, then set A is empty.

(v) The set of odd prime numbers is the empty set.

(vi) The set of squares of integers and the set of whole numbers are equal sets.

(vii) In n(P) = n(M), then P ↔ M.

(viii) If set P = set M, then n(P) = n(M).

(ix) n(A) = n(B) => A = B.

Solution:

(*i*) Set of even natural number less than 21

= {2,4,6,8,10,12,14,16,18,20}

∴ Cardinal Number of this set = 10

Set of odd natural numbers less than 21

= {1,3,5,7,9,11,13,15,17,19}

∴ Cardinal number of this set = 10

Now we see that cardinal numbers of both these sets = 10

∴ "The set of even natural numbers less than 21 and the set of odd natural numbers less than 21 are **equivalent** sets"………is a **True** statement. **Ans.**

(*ii*) E = {Factors of 16} = {1,2,4,8,16}

$1 \times 16 = 16$
$2 \times 8 = 16$
$4 \times 4 = 16$

F = {Factors of 20} = {1,2,4,5,10,20}

$1 \times 20 = 20$
$2 \times 10 = 20$
$4 \times 5 = 20$

Now we see that elements of set E and set F are not the same (identical)

∴ "If E = {Factors of 16} and F = {Factors of 20}, then E = F"……..is a **False** statement.

(*viii*) Set P = Set M

It means sets P and M are equal. Equal sets are equivalent also.

∴ Number of elements of set P = Number of elements of set M

∴ "If set P = set M, then *n* (P) = *n* (M)" ………is a **True** statement.

(*ix*) *n*(A) = *n*(B)

⇒ Number of elements of set A = Number of elements of set B

∴ Given sets are equivalent but not equal.

∴ "*n*(A) = *n*(B) ⇒ A = B" ……….is a **False** statement.

(*iii*) A = {Integers less than 20}

= {19,18,17,16,…..,0,–1,–2,–3,….}

∴ "The set A = {Integers less than 20} is a finite set"…..

……is a **False** statement.

(*iv*) A = {*x* : *x* is an even prime number} = {2}

∴ "If A = {*x* : *x* is an even prime number}, then set A is empty"….. is a **false** statement.

(*v*) Set of odd prime numbers

= {3,5,7,11,13,17,19,23,….}

∴ "The set of odd prime numbers is the empty set"…….is a **False** statement.

(*vi*)

Integer		Square of Integer	Whole No.
0	:	$(0)^2 = 0$	0
±1	:	$(\pm 1)^2 = 1$	1
±2	:	$(\pm 2)^2 = 4$	2
±3	:	$(\pm 3)^2 = 9$	3
±4	:	$(\pm 4)^2 = 16$	4
±5	:	$(\pm 5)^2 = 25$	5
…..	:	….. …..	…..
…..	:	….. …..	…..

∴ Set of squares of integers

= {0,1,4,9,16,25,…….}

Set of whole numbers = {0,1,2,3,4,5,6,7,….}

Hence "The set of squares of integers and the set of whole numbers are equal…**False** statement.

(*vii*) *n*(P) = *n*(M)

It means number of elements of set P

= Number of elements of set M.

∴ Sets P and M are equivalent.

∴ "If *n*(P) = *n*(M), then P ↔ M" … is a **True** Statement.

Question 4. Given, universal set = $\{x : x \in N, 10 \leq x \leq 35\}$.

A = $\{x \in N : x \leq 16\}$ and

B = $\{x : x > 29\}$ Find :

(*i*) A' (*ii*) B'.

Solution:

Universal set = $\{x : x \in N, 10 \leq x \leq 35\}$

= $\{10,11,12,13,14,15,....,34,35\}$

A = $\{x \in N, x \leq 16\}$

= $\{10,11,12,13,14,15,16\}$

B = $\{x : x > 29\}$

= $\{30,31,32,33,34,35\}$

(*i*) A' = $\{17,18,19,20,21,22,.....,33,34,35\}$

= $\{x : x \in N ; 17 \leq x \leq 35\}$

(*ii*) B' = $\{10,11,12,13,14,15,.....,29\}$

= $\{x : x \leq 29\}$

= $\{x : x \in N ; 10 \leq x \leq 29\}$

Question 5. Given universal set = $\{x \in Z : -6 < x \leq 6\}$, N = $\{n : n$ is a non-negative number$\}$ and

P = $\{x : x$ is a non-positive number$\}$

Find : (*i*) N' (*ii*) P'

Solution: Universal set = $\{x \in Z; -6 < x \leq 6\}$

= $\{-5,-4,-3,-2,-1,0,1,2,3,4,5,6\}$

N = $\{n : n$ is a non-negative number$\}$

= $\{0,1,2,3,4,5,6\}$

P = $\{x : x$ is a non-positive number$\}$

= $\{-5,-4,-3,-2,-1,0\}$

(*i*) N' = $\{-5,-4,-3,-2,-1\}$

(*ii*) P' = $\{1,2,3,4,5,6\}$

Question 6. If P = {factors of 36} and Q = {factors of 48}; find :

(*i*) $P \cup Q$ (*ii*) $P \cap Q$

(*iii*) $Q - P$ (*iv*) $P' \cap Q$.

$1 \times 36 = 36$ $\quad$ $1 \times 48 = 48$

$2 \times 18 = 36$ $\quad$ $2 \times 24 = 48$

$3 \times 12 = 36$ $\quad$ $3 \times 16 = 48$

$4 \times 9 = 36$ $\quad$ $4 \times 12 = 48$

$6 \times 6 = 36$ $\quad$ $6 \times 8 = 48$

$\therefore$ Factors of 36 = 1,2,3,4,6,9,12,18,36

Factors of 48 = 1,2,3,4,6,8,12,16,24,48

P = {factors of 36}

= {1,2,3,4,6,9,12,18,36}

Q = {Factors of 48}

= {1,2,3,4,6,8,12,16,24,48}

(*i*) $P \cup Q = \{1,2,3,4,6,9,12,18,36\} \cup \{1,2,3,4,6,8,12,16,24,48\}$

$= \{1,2,3,4,6,8,9,12,16,18,24,36,48\}$

(*ii*) $P \cap Q = \{1,2,3,4,6,9,12,18,36\} \cap \{1,2,3,4,6,8,12,16,24,48\}$

$= \{1,2,3,4,6,12\}$

(*iii*) $Q - P = \{1,2,3,4,6,8,12,16,24,48\} - \{1,2,3,4,6,9,12,18,36\}$

$= \{8,16,24,48\}$

(*iv*) $P' \cap Q$ = Only Q

$= Q - P$

$= \{1,2,3,4,6,8,12,16,24,48\} - \{1,2,3,4,6,9,12,18,36\}$

$= \{8,16,24,48\}$

Question 7. If $A = \{6,7,8,9\}$, $B = \{4,6,8,10\}$ and $C = \{x : x \in N : 2 < x \le 7\}$; find

(*i*) $A - B$ $\quad$ (*ii*) $B - C$

(*iii*) $B - (A - C)$ $\quad$ (*iv*) $A - (B \cup C)$

(*v*) $B - (A \cap C)$ $\quad$ (*vi*) $B - B$.

Solution:

Sol. $A = \{6,7,8,9\}$

$B = \{4,6,8,10\}$

$C = \{x : x \in N : 2 < x \le 7\}$

$= \{3,4,5,6,7\}$

(*i*) $A - B = \{6,7,8,9\} - \{4,6,8,10\}$

$= \{7,9\}$

(*ii*) $B - C = \{4,6,8,10\} - \{3,4,5,6,7\}$

$= \{8,10\}$

(*iii*) $A - C = \{6,7,8,9\} - \{3,4,5,6,7\}$

$= \{8,9\}$

$B - (A - C) = \{4,6,8,10\} - \{8,9\}$

$= \{4,6,10\}$

(*iv*) $B \cup C = \{4,6,8,10\} \cup \{3,4,5,6,7\}$

$= \{3,4,5,6,7,8,10\}$

$A - (B \cup C) = \{6,7,8,9\} - \{3,4,5,6,7,8,10\}$

$= \{9\}$

(*v*) $A \cap C = \{6,7,8,9\} \cap \{3,4,5,6,7\}$

$= \{6,7\}$

$B - (A \cap C) = \{4,6,8,10\} - \{6,7\}$

$= \{4,8,10\}$

(*vi*) $B - B = \{4,6,8,10\} - \{4,6,8,10\}$

$= \phi$

Question 8.

If $A = \{1,2,3,4,5\}$

$B = \{2,4,6,8\}$

and $C = \{3,4,5,6\}$

Verify :

(*i*) $A - (B \cup C) = (A-B) \cap (A-C)$

(*ii*) $A - (B \cap C) = (A-B) \cup (A-C)$

Solution:

$A = \{1,2,3,4,5\}$

$B = \{2,4,6,8\}$

$C = \{3,4,5,6\}$

(*i*) $B \cup C = \{2,4,6,8\} \cup \{3,4,5,6\}$

$= \{2,3,4,5,6,8\}$

$A - (B \cup C) = \{1,2,3,4,5\} - \{2,3,4,5,6,8\}$

$= \{1\}$

$A - B = \{1,2,3,4,5\} - \{2,4,6,8\}$

$= \{1,3,5\}$

$A - C = \{1,2,3,4,5\} - \{3,4,5,6\}$

$= \{1,2\}$

$\therefore (A-B) \cap (A-C) = \{1,3,5\} — \{1,2\} = \{1\}$

$\therefore A - (B \cup C) = (A-B) \cap (A-C)$

(*ii*) $B \cap C = \{2,4,6,8\} \cap \{3,4,5,6\}$

$= \{4,6\}$

$A - (B \cap C) = \{1,2,3,4,5\} - \{4,6\}$

$= \{1,2,3,5\}$

$A-B = \{1,2,3,4,5\} - \{2,4,6,8\}$

$= \{1,3,5\}$

$A-C = \{1,2,3,4,5\} - \{3,4,5,6\}$

$= \{1,2\}$

$(A-B) \cup (A-C) = \{1,3,5\} \cup \{1,2\}$

$= \{1,2,3,5\}$

$\therefore A - (B \cap C) = (A-B) \cup (A-C)$

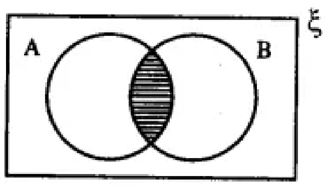
Question 9.

Draw a Venn-diagram to show the relationship between two overlapping sets A and B. Now shade the region representing :

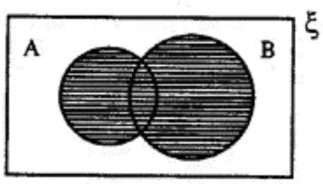
(*i*) $A \cap B$ (*ii*) $A \cup B$

(*iii*) $B - A$

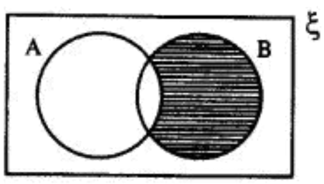
Solution: (*i*) $A \cap B =$



(*ii*) $A \cup B =$



(*iii*) $B - A =$

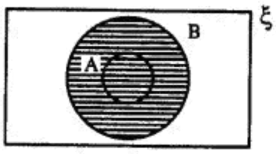


Question 10. Draw a Venn-diagram to show the relationship between two sets A and B ; such that A $\subseteq$ B, Now shade the region representing :

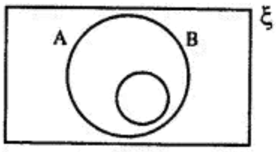
(*i*) $A \cup B$ (*ii*) $B' \cap A$

(*iii*) $A \cap B$ (*iv*) $(A \cup B)'$

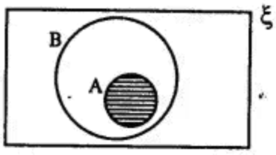
Solution: (*i*) $A \cup B$ =



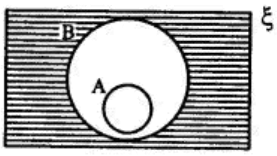
(*ii*) $B' \cap A$ =



(*iii*) $A \cap B$ =



(*iv*) $(A \cup B)'$ =



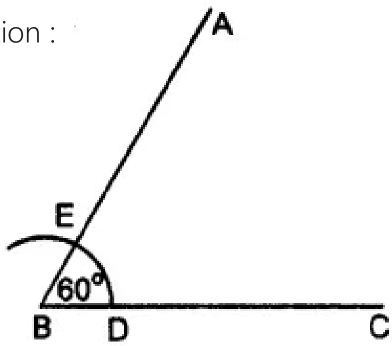
CONSTRUCTION
[CHAPTER-19]

Question 1.

Draw a line segment BC = 4 cm. Construct angle ABC = 60°.

Solution:

Steps of Construction :



1. Draw a line segment BC = 4 cm
2. With B as centre, draw an arc of any suitable radius which cuts BC at the point D.
3. With D as centre, and the same radius as in step 2, draw one more arc which cuts the previous arc at the point E.
4. Join BE and produce it to the point A.

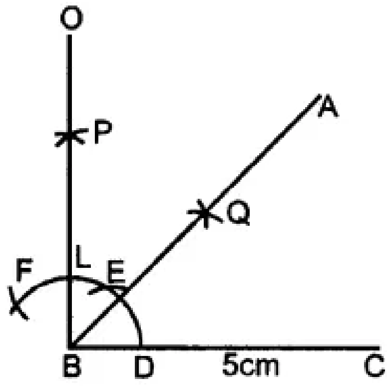
Thus $\angle ABC = 60°$

Question 2.

Construct angle ABC = 45° in which BC = 5 cm and AB = 4.6 cm.

Solution:

Steps of Construction :



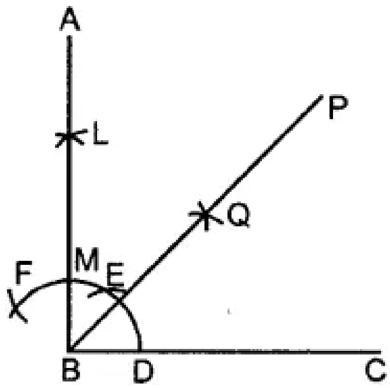
1. Draw a line segment BC = 5 cm
2. Taking B as centre, draw an arc of any suitable radius, which cuts BC at the point D.
3. With D as centre and the same radius, as taken in step 2, draw an arc which cuts the previous arc at point E.
4. With E as centre and the same radius, draw one more arc which cuts the
5. With E and F as centres and radii equal to more than half the distance between E at F, draw arc which cut each other at point P.
6. Join BP to meet EF at L and produce to point O. Then ∠OBC = 90°
7. Draw BA, the bisector of angle OBC. [With D, L as centres and suitable radius draw two arc meeting each other at Q produced it to R]

   => ∠ABC = 45° [∴ BA is bisector of ∠ OBC ∴ ∠ ABC = = 45°]
8. From BR cut arc AB = 4.6 cm

Question 3.

Construct angle ABC = 90°. Draw BP, the bisector of angle ABC. State the measure of angle PBC.

Solution:



1. Draw ∠ABC = 90° (as in Ques. 4)
2. Draw bisector of ∠ABC

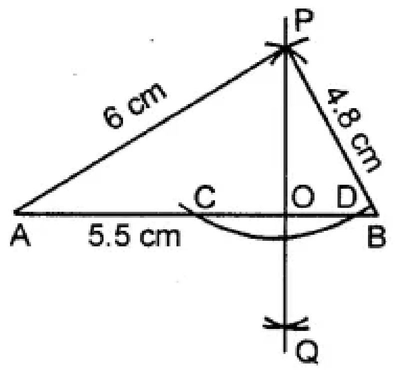
   Then ∠PBC = (90°) = 45°

Question 4.

Draw a line segment AB = 5.5 cm. Mark a point P, such that PA = 6 cm and PB = 4.8 cm. From the point P, draw perpendicular to AB.

Solution:

Step of Construction :



1. Draw a line segment AB = 5.5 cm
2. With A as centre and radius = 6 cm, draw an arc.
3. With B as centre and radius = 4.8 cm draw another arc.
4. Let these arcs meet each other at the point P.
   PA = 6 cm, PB = 4.8
5. With P as centre and some suitable radius draw an arc meeting AB at the points C and D.
6. With C as centre and radius more than half of CD, draw an arc.
7. With D as centre and same radius as in step 6, draw an arc.
8. Let these arcs meet each other at the point Q.
9. Join PQ.
10. The PQ meet AB at point O.
    Then PO $\perp$ AB i.e; $\angle AOP = 90° = \angle POB$.

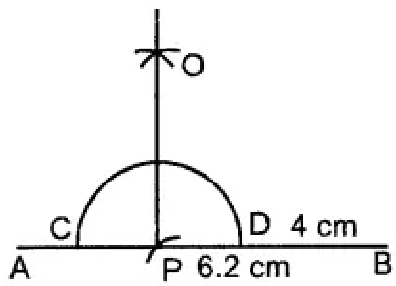
Question 5.

Draw a line segment AB = 6.2 cm. Mark a point P in AB such that BP = 4 cm. Through point P draw perpendicular to AB.

Solution:

Steps of Construction :

1. Draw a line segment AB = 6.2 cm
2. Cut off BP = 4 cm
3. With P as centre and some radius draw arc meeting AB at the points C, D.
4. With C, D as centres and equal radii [each is more than half of CD] draw two arcs, meeting each other at the point O.
5. Join OP. Then OP is perpendicular for AB.

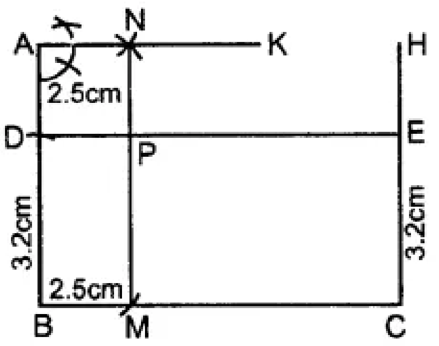


Question 6.

Construct an angle ABC = 90°. Locate a point P which is 2.5 cm from AB and 3.2 cm from BC.

Solution:

Steps of construction :

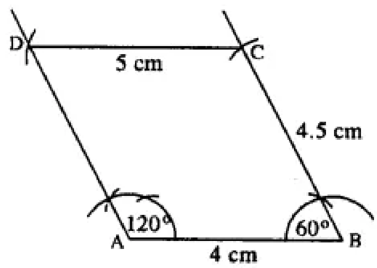


1. Draw ∠ABC = 90°
2. From AB, cut BD = 3.2 cm.
3. Through point C, draw CH ⊥BC. From CH, cut CE = 3.2. Join DE. Now DE is a line parallel to BC and at a distance of 3.2 cm from BC.
4. From BC cut BM = 2.5 cm.
5. Through point A, draw AK⊥ AB. From AK cut AN = 2.5 cm. Join NM. Therefore NM is parallel to AB and at a distance of 2.5 cm from AB.
6. DE and MN intersect each other at P. Thus P is the required point which is 2.5 cm from AB and 3.2 cm from BC.

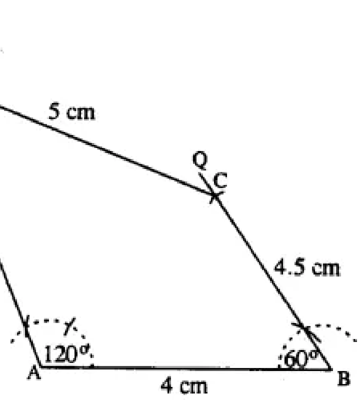
Question 7.

Construct a quadrilateral ABCD in which ; ∠A = 120°, ∠B = 60°, AB = 4 cm, BC = 4.5 cm and CD = 5 cm.

Solution:



Actual figure is constructed as follow



Steps :

1. Draw AB = 4 cm.
2. At A, draw ∠PAB = 120°.
3. At B, draw ∠QBA = 60°.

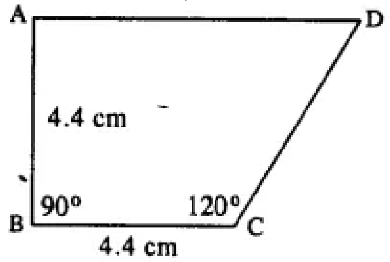
4. From BQ, cut BC = 4.5 cm.
5. From C, draw an arc of radius 5 cm which meets AP at D.
6. Join CD.

Thus ABCD is the required quadrilateral.

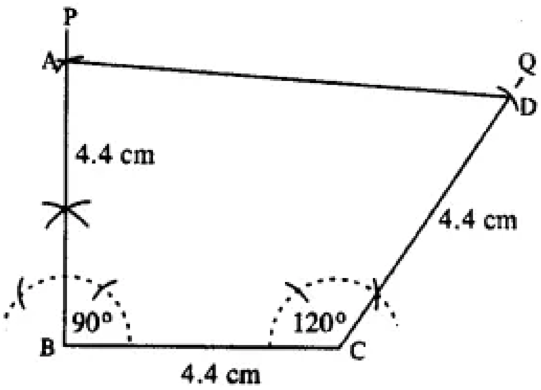
Question 8.

Construct a quadrilateral ABCD, such that AB = BC = CD = 4.4 cm, ∠B = 90° and ∠C = 120°.

Solution:



Actual figure is constructed as follow :



Steps :

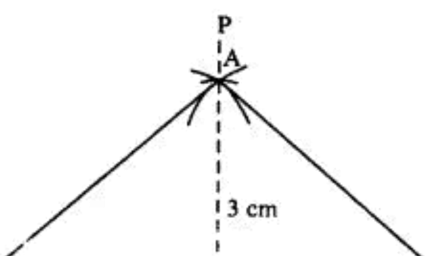
1. Draw BC = 4.4 cm.
2. At B, draw ∠PBC = 90°.
3. Cut BA = 4.4 cm.
4. At C, draw ∠QCB = 120°.
5. Cut CD = 4.4 cm.
6. Join AD.

Thus ABCD is the required quadrilateral.

Question 9.

Using ruler and compasses only, construct a rhombus whose diagonals are 8 cm and 6 cm. Measure the length of its one side.

Solution:



Steps :

1. Draw BD = 8 cm.
2. Draw perpendicular bisector PQ of BD.
3. Cut OA = OC = 3 cm [half the diagonal 6 cm]
4. Join AB, AD, BC and CD.
5. Measure side AB which is 5 cm.

Thus ABCD is the required rhombus.

REPRESENTING 3-D IN 2-D
[CHAPTER-18]

Question 1.

Can a polyhedron have 8 faces, 26 edges and 16 vertices?

Solution:

Number of faces = 8

Number of vertices = 16

Number of edges = 26

Using Euler's formula

F + V – E

? 8 + 16 – 26 ≠ -2

? -2 ≠ 2

No, a polyhedron cannot have 8 faces, 26 edges and 16 vertices.

Question 2.

Can a polyhedron have: (i) 3 triangles only ?

(ii) 4 triangles only ?

(iii) a square and four triangles ? Solution:

(i) No.

(ii) Yes. (iii) Yes.

Question 3.

What is the least number of planes that can enclose a solid? What is the name of the solid.
Solution:
The least number of planes that can enclose a solid is 4.

The name of the solid is Tetrahedron.

Question 4.

Is a square prism same as a cube?

Solution:
Yes, a square prism is same as a cube.

DATA HANDLING

[CHAPTER-25 and 26]

Statistics (Chap – 25)

Question 1.
Following are the marks obtained by 30 students in an examinations.

15	20	8	9	10
16	17	20	24	30
44	47	38	36	40
27	25	28	30	19
7	11	21	31	41
37	47	23	20	17

Taking class intervals 0-10, 10-20, ……… 40-50 ; construct a frequency table.
Solution:

Class Intervals	Tally Marks	Frequency
0 – 10	𝍷𝍷𝍷	3
10 – 20	𝍸 𝍷𝍷	7
20 – 30	𝍸 𝍷𝍷𝍷𝍷	9
30 – 40	𝍸 𝍷	6
40 - 50	𝍸	5

Question 2.
Construct a frequency distribution table for the following data ; taking class-intervals 4-6, 6-8, ……… 14-16.

11·5 6·3 7·8 9·2 10·5 4·5, 6 8·3 12·5 15·8
7·4 5·3 8·4 15·2 8·9 9·8 8·25 6·5 5·8 10·5
4·6 6·4 8·9 10·8 12·7 14·2 15·3 11·7 9·9 8·8
6·6 4·3 4·7 9·4 10·1 15·5 14·4 12·2 7·7 5·5

Solution:

Class Intervals	Tally Marks	Frequency
4–6	卌 II	7
6–8	卌 III	8
8–10	卌 卌	10
10–12	卌 I	6
12–14	III	3
14–16	卌 I	6

Question 3. Fill in

(i)Lower class limit of 15-18 is ....

(ii) Upper class limit of 24-30 is ....

(iii) Upper limit of 5-12.5 is ....

(iv) If the upper and the lower limits of a class interval are 16 and 10 ; the class-interval is ....

(iii) If the lower and the upper limits of a class interval are 7.5 and 12.5 ; the class interval is ....

Solution:

(i) Lower class limit of 15 – 18 is 15.

(ii) Upper class limit of 24 – 30 is 30.

(iii) Upper limit of 5 – 12.5 is 12.5

(iv) If the upper and lower limits of a class interval are 16 and 10 ; the class interval is 10 – 16

(v) If the lower and upper limits of a class interval are 7.5 and 12.5 ; the class interval is 7.5 – 12.5

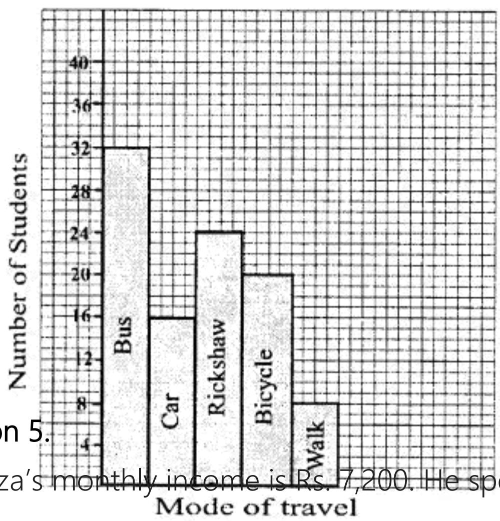
Graphical Representation of Data (Chap - 26)

Question 4.

Hundred students from a certain locality use different modes of travelling to school as given below. Draw a bar graph.

Bus	Car	Rickshaw	Bicycle	Walk
32	16	24	20	8

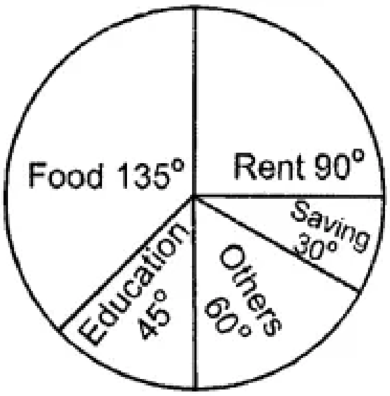
Solution:



Question 5.

Mr. Mirza's monthly income is Rs. 7,200. He spends Rs. 1,800 on rent, Rs. 2,700 on food, Rs. 900 on

education of his children ; Rs. 1,200 on Other things and saves the rest.
Draw a pie-chart to represent it.
Solution:



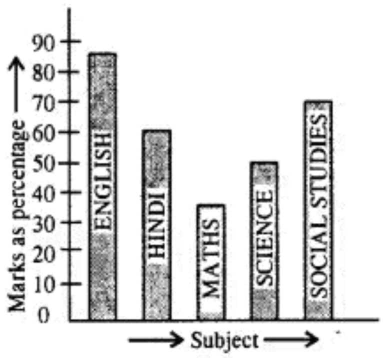
Name of items	Expenditure/Saving in Rupees	Central angle
Rent	1800	$\frac{1800}{7200} \times 360^{\circ} = 90^{\circ}$
Food	2700	$\frac{2700}{7200} \times 360^{\circ} = 135^{\circ}$
Education	900	$\frac{900}{7200} \times 360^{\circ} = 45^{\circ}$
Others	1200	$\frac{1200}{7200} \times 360^{\circ} = 60^{\circ}$
Saving	600	$\frac{600}{7200} \times 360^{\circ} = 30^{\circ}$
Total	7200	360°

Question 6.

The percentage of marks obtained, in different subjects by Ashok Sharma (in an examination) are given below. Draw a bar graph to represent it.

English	Hindi	Maths	Science	Social Studies
85	60	35	50	70

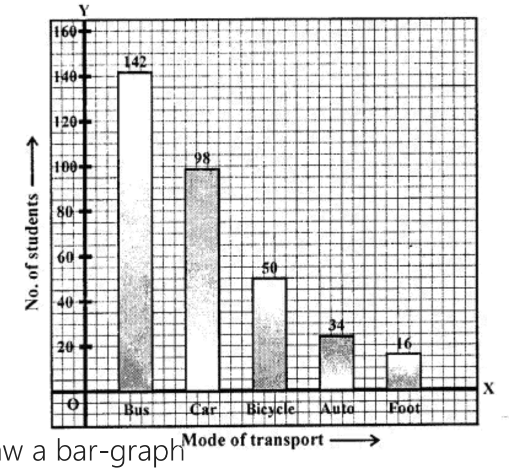
Solution:



Question 7.
Students of a small school use different modes of travel to school as shown below:

Mode	Bus	Car	Bicycle	Auto	On foot
No. of students	142	98	50	34	16

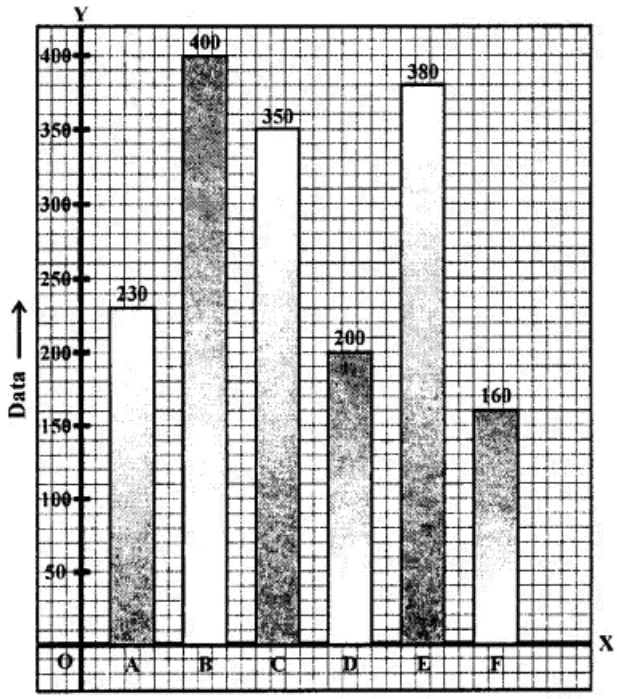
Draw a suitable bar graph.



Question 8.
For the following table, draw a bar-graph

A	B	C	D	E	F
230	400	350	200	380	160

Solution:



Question 9. Manoj appeared for ICSE examination 2018 and secured percentage of marks as shown in the following table:

Subject	Hindi	English	Maths	Science	Social Study
Marks as percent	60	45	42	48	75

Represent the above data by drawing a suitable bar graph.

Solution:

Question 10. For the data given above in question number 9, draw a suitable pie-graph.

Solution:

60 + 45 + 42 + 48 + 75 = 270

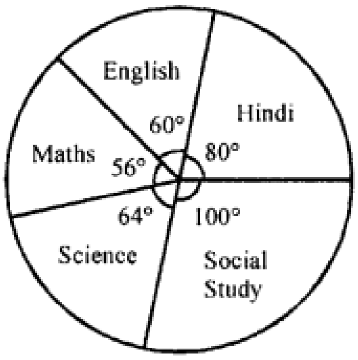
$\therefore$ Central angle for Hindi $= \frac{60}{270} \times 360° = 80°$

Central angle for English $= \frac{45}{270} \times 360° = 60°$

Central angle for Maths $= \frac{42}{270} \times 360° = 45°$

Central angle for Science $= \frac{48}{270} \times 360° = 64°$

and Central angle for Social study $= \frac{75}{270} \times 360° = 100°$



PROBABILITY
[CHAPTER – 27]

Question 1.
A coin is tossed twice. Find the probability of getting:
(i) exactly one head (ii) exactly one tail
(iii) two tails (iv) two heads
Solution:

(*i*) Exactly one head
Possible number of favourable outcomes = 2
(*i.e.* TH and HT)
Total number of possible outcomes = 4

$$\therefore \ P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

(*ii*) Exactly one tail
Possible number of favourable outcomes = 2
(*i.e.* TH and HT)
Total number of possible outcomes = 4

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

(*iii*) Two tails
Possible number of favourable outcomes = 1
(*i.e.* TT)
Total number of possible outcomes = 4

$$\therefore \ P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{1}{4}$$

(*iv*) Two heads
Possible number of favourable outcomes = 1
(*i.e.* HH)
Total number of possible outcomes = 4

$$\therefore \ P(E) = \frac{1}{4}$$

Question 2.

A letter is chosen from the word 'PENCIL' what is the probability that the letter chosen is a consonant?

Solution:

Total no. of letters in the word 'PENCIL = 6 Total Number of Consonant = 'PNCL' i.e. 4

$$P(E) = \frac{\text{Total No. of consonants}}{\text{Total No. of letters in the word PENCIL}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

Question 3.

A bag contains a black ball, a red ball and a green ball, all the balls are identical in shape and size. A ball is drawn from the bag without looking into it. What is the probability that the ball drawn is:

(i) a red ball

(ii) not a red ball

(iii) a white ball.

Solution:

Total number of possible outcomes = 3

$$\therefore P(E) = \frac{1}{3}$$

(*ii*) Not a red ball

Number of favourable outcomes

= Green ball + Black ball

= 1 + 1 + 2

$$\therefore P(E) = \frac{2}{3}$$

(*iii*) A white ball

Number of favourable outcomes = 0

$$\therefore P(E) = \frac{0}{3} = 0$$

Question 4.

In a single throw of a die, find the probability of getting a number

(i) greater than 2

(ii) less than or equal to 2

(iii) not greater than 2.

Solution:

A die has six numbers = 1, 2, 3, 4, 5, 6

Number of possible outcomes = 6

$$\therefore P(E) = \frac{4}{6} = \frac{2}{3}$$

(*ii*) Less than or equal to 2

Number of favourable outcomes = 1, 2

$$\therefore P(E) = \frac{2}{6} = \frac{1}{3}.$$

(*iii*) Not greater than 2

Number of favourable outcomes = 1, 2

$$\therefore P(E) = \frac{2}{6} = \frac{1}{3}$$

Question 5.

A bag contains 3 white, 5 black and 2 red balls, all of the same shape and size. A ball is drawn from the bag without looking into it, find the probability that the ball drawn is:

(i) a black ball.

(ii) a red ball.

(iii)a white ball.

(iv) not a red ball.

(v) not a black ball.

Solution:

In a bag, 3 balls are white
2 balls are red
5 balls are black
Total number of balls = 3 + 2 + 5 = 10

(i) Number of possible outcome of one black ball = 10
and number of favouable outcome of one black ball = 5

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{5}{10} = \frac{1}{2}$$

(*ii*) Number of possible outcome of one red ball = 10
and number of favourable outcome = 2

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{2}{10} = \frac{1}{5}$$

(*iii*) Number of possible outcome of white ball = 10
and number of favourable outcome = 3

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{10}$$

(*iv*) Number of possible outcome = 10
Number of favourable outcome
= 3 + 5 = 8
not a red ball

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{8}{10} = \frac{4}{5}$$

(*v*) Number of possible outcomes = 10
Number of favourable outcome
not a black ball = 3 + 2 = 5

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{5}{10} = \frac{1}{2}$$

Question 6.

A book contains 92 pages. A page is chosen at random. What is the probability that the sum of the digits in the page number is 9?

Solution:

Number of pages of the book = 92

Which are from 1 to 92

Number of possible outcomes = 92

? Number of pages whose sum of its page is 9 = 10

i.e. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 $\therefore P(E) = \frac{10}{92} = \frac{5}{46}$

Question 7. Two coins are tossed together. What is the probability of getting:

(i) at least one head

(ii) both heads or both tails.

Solution:

(*i*) At least one head, then

Number of outcomes = 3

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

$$= \frac{3}{4}$$

(*ii*) When both head or both tails, then

Number of outcomes = 2

$$\therefore P(E) = \frac{\text{Number of favourable outcome}}{\text{Number of all possible outcome}}$$

Number of coins = 2 x 2 = 4

which are HH, HT, TH, TT

$$= \frac{2}{4} = \frac{1}{2}$$

Question 8. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is:

(i) 0 (ii) 12 (iii) less than 12

(iv) less than or equal to 12

Solution:

Total outcomes = 36 i.e.

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(*i*) Favourable outcomes = 0

$$P(E) = \frac{0}{36} = 0$$

(*ii*) Favourable outcomes = 1 *i.e.* (6, 6)

$$P(E) = \frac{1}{36}$$

(*iii*) Favourable outcomes = 35 [Except (6, 6)]

$$P(E) = \frac{35}{36}$$

(*iv*) Favourable outcomes = 36

$$P(E) = \frac{36}{36} = 1$$

Question 9. A die is thrown once. Find the

probability of getting:
(i) a prime number
(ii) a number greater than 3
(iii) a number other than 3 and 5
(iv) a number less than 6
(v) a number greater than 6.
Solution:

Total outcomes = 6

i.e., 1, 2, 3, 4, 5 and 6

(*i*) Favourable outcomes = 3 *i.e.*, 2, 3, 5

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

(*ii*) Favourable outcomes = 3 *i.e.*, 4, 5, 6

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

(*iii*) Favourable outcomes = 4 *i.e.*, 1, 2, 4, 6

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

(*iv*) Favourable outcomes = 5
i.e., 1, 2, 3, 4, and 5

$$P(E) = \frac{5}{6}$$

(*v*) Favourable outcomes = 0

$$P(E) = \frac{0}{6} = 0$$

Question 10.

Two coins are tossed together. Find the probability of getting:
(i) exactly one tail
(ii) at least one head
(iii) no head
(iv) at most one head
Solution:

Total outcomes = 4

i.e., HH, HT, TT, TH

(*i*) Favourable outcomes = 2 *i.e.*, HT and TH

$$P(E) = \frac{2}{4} = \frac{1}{2}$$

(*ii*) Favourable outcomes = 3
i.e., HH, HT and TH

$$P(E) = \frac{3}{4}$$

(*iii*) Favourable outcomes = 1 *i.e.*, TT

$$P(E) = \frac{1}{4}$$

(*iv*) Favourable outcomes = 3
i.e., HH, HT and TH

$$P(E) = \frac{3}{4}$$