Model QUESTION BANK

First Mock test

CLASS – X

MATHEMATICS

Goods and service tax

[CHAPTER – 1]

Example 1: A shopkeeper sells some edible oil for ₹7200 at its MP. The shopkeeper pays GST of ₹120 to the Government. If the GST charged throughout is 5%, calculate the price paid by the shopkeeper for the oil inclusive of tax.

Solution:

$$\text{GST} = 5\% \text{ of Profit}$$

$$₹120 = \frac{5}{100} \times \text{Profit}$$

$$\text{Profit} = ₹\frac{120 \times 100}{5} = ₹2400$$

$$\text{CP} = \text{SP} - \text{Profit}$$

$$\text{CP} = ₹(7200 - 2400) = ₹4800$$

Tax paid at the time of buying = 5% of ₹4800

= ₹240

Price paid by the shopkeeper = ₹(4800 + 240)

= ₹5040

Example 2: When some food was ordered from a restaurant, the bill excluding tax was ₹750. If GST charged is 5%, find the total amount paid by the consumer.

Solution:

Amount of GST = 5% of the bill

$$= \frac{5}{100} \times ₹750 = ₹37.50$$

Amount paid by the consumer

$$= ₹(750 + 37.50) = ₹787.50$$

Note: here SGST and CGST are 2.5% each

$$= \frac{₹37.50}{2} = ₹18.75$$

Example 3: When Mr. Mukherjee stayed in a hotel for 2 days he had to pay ₹7080 including 18% GST. What is the tariff of the hotel for a unit of accommodation?

Solution:

Let the original amount of the bill before tax be ₹x

$$\therefore \quad x + \frac{18}{100}x = 7080$$

$$\frac{118x}{100} = 7080$$

$$x = \frac{7080}{118} \times 100 = ₹6000$$

Tariff of the hotel per day $= ₹\frac{6000}{2} = ₹3000$

Example 4: **One kg of ghee costs ₹588 which includes 12% GST. Find the price of 1 kg of ghee before tax and the amount of tax on it.**

Solution:

Let the price of ghee per kg be ₹x

$$\therefore \quad x + \frac{12}{100}x = 588$$

$$\frac{112x}{100} = 588$$

$$x = \frac{588 \times 100}{112} = 525$$

$\therefore$ Cost of 1 kg of ghee = ₹525

Amount of GST = ₹(588 – 525) = ₹63

Ans.

Example 5: A manufacturer produces a sewing machine for ₹5000 and sells it to the wholesaler for ₹5500. The wholesaler sells it to a dealer for ₹7000 and the dealer sells it to a shopkeeper for ₹7500. The shopkeeper makes a profit of ₹1000 by selling to the consumer. If GST charged at each stage is 12%, find

(*i*) The GST paid by the manufacturer and wholesaler to the Government.

(*ii*) The final price paid by the consumer.

Solution:

(*i*) GST deposited by the manufacturer

$= 12\% \text{ of SP } = \frac{12}{100} \times ₹5500 = ₹660$

GST deposited by the wholesaler
= 12% of his profit

$= \frac{12}{100} \times ₹(7000 - 5500) = ₹180$

GST deposited by the dealer
= 12% of his profit

$= \frac{12}{100} \times ₹(7500 - 7000) = ₹60$

GST deposited by the shopkeeper

$= 12\% \text{ of his profit } = \frac{12}{100} \times ₹1000 = ₹120$

Total GST deposited by manufacturer and wholesaler = ₹(660 + 180) = ₹840

(*ii*) Price paid by the consumer

= SP + 12% tax charged by the shopkeeper

$= ₹8500 + \frac{12}{100} \times ₹8500 = ₹(8500 + 1020) = ₹9520$

Example 6: A shopkeeper buys certain quantity of cashew nuts for ₹7200 and sells it to a consumer for ₹9000. If the rate of GST is 5%, find the GST paid by the shopkeeper to the Government.

Solution:

Output Tax = 5% of SP $= \frac{5}{100} \times ₹9000 = ₹450$

Input Tax = 5% of CP $= \frac{5}{100} \times ₹7200 = ₹360$

GST deposited by the shopkeeper

= ₹(450 – 360) = ₹90 with the Govt.

Example 7: Vamsi buys a camera for ₹8000 and sells it for ₹10,000. GST charged is 28%.

(*i*) Find the GST deposited by him with the Government.

(*ii*) What is the price paid by the customer?

Solution

(*i*) GST paid by Vamsi

= 28% of ₹(10,000 – 8000)

$= \frac{28}{100} \times ₹2000 = ₹560$

(*ii*) Price paid by the customer

$= ₹\left(10,000 + \frac{28}{100} \times 10,000\right) = ₹12800$

Example 8: A shopkeeper buys a printer at a discount of 30% on the marked price of ₹8000. He sells the printer to a customer at marked price. GST charged at each stage is 18%. Find

(*i*) GST paid by the shopkeeper to the Government.

(*ii*) The price paid by the shopkeeper for the article inclusive of tax.

(*iii*) The cost to the customer inclusive of tax.

Solution:

(*i*) Cost to the shopkeeper with 30% discount

$= 70\% \text{ of MP } = \frac{70}{100} \times ₹8000 = ₹5600$

SP charged by the shopkeeper = ₹8000

$\therefore$ His profit = ₹(8000 – 5600) = ₹2400

GST deposited by him = 18% of ₹2400 = ₹432

(*ii*) Price paid by the shopkeeper = Cost + Tax

$= ₹5600 + \frac{18}{100} \times ₹5600$

= ₹(5600 + 1008) = ₹6608

(*iii*) Price paid by the customer

= Printed Price + 18% tax on it

$= ₹8000 + \frac{18}{100} \times ₹8000 = ₹9440$

Example 9: A manufacturer marks a mobile for ₹6000. He sells it to the wholesaler at 25% discount. The wholesaler sells it to a retailer at 20% discount on MP. If the retailer sells it at MP and GST charged is 12% at every stage Find the GST paid to the Government by

(*i*) The wholesaler (*ii*) The retailer.

Solution:

(*i*) Cost price to the wholesaler after 25% discount

$= 75\% \text{ of MP } = \frac{75}{100} \times ₹6000 = ₹4500$

Selling price after giving 20% discount

$= 80\% \text{ of MP } = \frac{80}{100} \times ₹6000 = ₹4800$

Profit = ₹300

GST paid by him = 12% of ₹300 = ₹36

(*ii*) Cost price to the retailer = ₹4800

Selling price = ₹6000

Profit = ₹1200

GST paid by retailer 12% of Rs.1200 = Rs.144

Example 10: A manufacturer sells a TV set for ₹20,000 to a wholesaler, who sells it to a retailer at a profit of 10%. The retailer sells it to a customer at 15% profit. If the rate of GST at each stage is 28%, find the

(*i*) GST paid by the wholesaler to the govt.

(*ii*) Price paid by the retailer including tax.

(*iii*) Total GST received by the govt.

(*iv*) Price paid by the customer

Solution:

SP for the manufacturer = CP for the wholesaler

= ₹20,000

Profit of wholesaler = 10% of ₹20000 = ₹2000

SP for the wholesaler = ₹22000

= CP for the retailer

Now Profit of retailer = 15% of ₹22000 = ₹3300

∴ SP for the retailer = ₹(22000 + 3300) = ₹25300

(*i*) Wholesaler's GST = 28% of his profit

$$= \frac{28}{100} \times ₹\,2000 = ₹560$$

(*ii*) Price paid by the retailer

$$= ₹\,22000 + \frac{28}{100} \times ₹\,22000$$

= ₹22000 + ₹6160

= ₹28160

(*iii*) Total GST received by the Govt.

= 28% of SP of the retailer

$$= \frac{28}{100} \times ₹\,25300 = ₹7084$$

(*iv*) Price paid by the customer

= SP + Total GST

= ₹25300 + ₹7084 = ₹32,384

Banking

[CHAPTER – 2]

Example 1

Mrs. Singh invests ₹ 250 every month for 24 months in a bank and collects ₹ 6,750 at the end of the term. Find the rate of simple interest paid by the bank on this recurring deposit.

Solution:

Let r% be the rate of simple interest

Given P = ₹ 250, n = 24 months

The period of recurring deposit $= N = \frac{n(n+1)}{24} = \frac{24 \times 25}{24} = 25$ years

Since the amount deposited with the bank is ₹ 250 × 24 = ₹ 6,000, the interest is therefore

$$= \frac{PNr}{100} = \frac{250 \times 25 \times r}{100} = \frac{250}{4}r = \frac{125}{2}r$$

Thus $6{,}000 + \frac{125}{2}r = 6{,}750 \Rightarrow \frac{125}{2}r = 750 \Rightarrow r = 12$

∴ 12% is the rate of interest. **Ans.**

Example 2

Samir deposits ₹ 600 per month in a recurring deposit account for 2 years. If he receiv ₹ 15,450 at the time of maturity, find the rate of interest p.a. pays by the bank.

Solution:

Given; Monthly instalment (P) = ₹ 600

time = n = 24 months

Let r% be the rate of interest p.a.

$$\text{S.I.} = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$= ₹\ 600 \times \frac{24 \times 25}{2 \times 12} \times \frac{r}{100} = ₹\ 150r$$

Amount = ₹ 600 × 24 + S.I.

⇒ ₹ 15,450 = ₹ 14,400 + ₹ 150r

⇒ ₹ 150r = ₹ 15,450 – ₹ 14,400

⇒ 150r = 1050

$$r = \frac{1050}{150} = 7\%$$

∴ rate of interest = 7% **Ans.**

Example 3

Mr. S. Mukherjee has a recurring deposit account in State Bank of India of ₹ 2,000 per month at the rate of 10% p.a. If he gets ₹ 83,100 at the time of maturity find the total time for which the account was held.

Solution:

Monthly instalment = ₹ 2,000

Rate of interest = 10% per annum.

Let the time period of recurring account = x months.

Interest = ₹ $2000 \times \frac{x(x+1)}{2 \times 12} \times \frac{10}{100} = \frac{25x(x+1)}{3}$

Total amount deposited in x months = ₹ 2,000 × x = ₹ 2000x

$\therefore$ Amount received at maturity = ₹ $2{,}000x + \frac{25x(x+1)}{3}$

By the given condition; $2000x + \frac{25(x^2+x)}{3} = 83100$

$\Rightarrow 6000x + 25x^2 + 25x = 249300$

$\Rightarrow 25x^2 + 6025x = 249300$

$\Rightarrow 25x^2 + 6025x - 249300 = 0$

$\Rightarrow x^2 + 241x - 9972 = 0$ [Dividing both sides by 25]

$\Rightarrow x^2 + 277x - 36x - 9972 = 0$

$\Rightarrow x(x+277) - 36(x+277) = 0$

$\Rightarrow (x + 277)(x - 36 = 0$

$\therefore x = 36$ or -277

Since the time period X month cannot be negative,

Therefore Time = 36 month = 3 years.

Shares and dividend

[CHAPTER – 3]

Example 1

By investing ₹ 7,500 in a company paying 10 per cent dividend, an income of ₹ 500 is received. What price is paid for each ₹ 100 share? **[ICSE 1990]**

Solution:

For value of each share = ₹ 100

Dividend per share = ₹ $100 \times \frac{10}{100}$ = ₹ 10

Total income = ₹ 500

$\therefore$ No. of shares = $\frac{500}{10} = 50$

Total investment = ₹ 7,500

$\therefore$ Cost price of each share = ₹ $\frac{7,500}{50}$ = ₹ 150 **Ans.**

Example 2 .

A man sold 400 (₹ 20) shares, paying 5% at ₹ 18 and invested the proceeds in (₹ 10) shares, paying 7% at ₹ 12. How many (₹ 10) shares did he buy and what was the change of income?

Solution:

Dividend per (₹ 20) share = ₹ $20 \times \frac{5}{100}$ = ₹ 1

$\therefore$ dividend from 400 (₹ 20) shares = ₹ 1 × 400 = ₹ 400

S.P. of 400 (₹ 20) shares at ₹ 18 each = ₹ 18 × 400 = ₹ 7,200

$\therefore$ No. of (₹ 10) shares purchased at ₹ 12 each = $\frac{7,200}{12} = 600$

Dividend per (₹ 10) share = ₹ $10 \times \frac{7}{100}$ = ₹ $\frac{7}{10}$

$\therefore$ dividend from 600 (₹ 10) shares = ₹ $\frac{7}{10} \times 600$ = ₹ 420

$\therefore$ change in income = ₹ 420 – ₹ 400 = ₹ 20 (gain). **Ans.**

Example 3 .

An investor holds 2,000 shares of a company that have a face value of ₹ 100 each. Th company pays a 25% dividend annually. Calculate

(i) The annual dividend.

(ii) The percentage return (correct to the nearest integer) if the shares were bought at 40% premium.

Solution:

(i) Annual dividend

$= 25\%$ total N.V.

$= \frac{25}{100} \times 2{,}000 \times 100 = ₹\,50{,}000$. **Ans.**

(ii) Money invested to buy 2,000 shares

$= ₹\,2{,}000 \times 140$

$\therefore$ Percentage return $= \frac{\text{Dividend}}{\text{Money Invested}} \times 100 = \frac{50{,}000}{2{,}000 \times 140}$

$= 17.86 = 18\%$ (correct to the nearest integer). **Ans.**

Example 4 .

A person invested 20%, 30% and 25% of his savings in buying shares of three different companies A, B and C which declared dividends of 10%, 12% and 15% respectively. If his total income on account of dividends be ₹ 2,337.50, find his savings and the amount which he invested buying shares of each company.

Solution:

Le the savings be ₹ x.

Investment in Company;

$$A = \frac{20}{100} \times ₹\,x = ₹\,\frac{x}{5},\quad B = \frac{30}{100} \times ₹\,x = ₹\,\frac{3x}{10},\quad C = \frac{25}{100} \times ₹\,x = ₹\,\frac{x}{4}$$

Dividends from Companies:

$$A = \frac{10}{100} \times ₹\,\frac{x}{5} = ₹\,\frac{x}{50},\quad B = \frac{12}{100} \times ₹\,\frac{3x}{10} = ₹\,\frac{9x}{250},\quad C = \frac{15}{100} \times ₹\,\frac{x}{4} = ₹\,\frac{15x}{400}$$

But the total dividend $= ₹\,2{,}337.50$

$$\therefore ₹\left(\frac{x}{50} + \frac{9x}{250} + \frac{15x}{400}\right) = ₹\,2{,}337.50$$

$$\Rightarrow \frac{40x + 72x + 75x}{2000} = ₹\,2{,}337.50 \Rightarrow \frac{187x}{2000} = ₹\,2{,}337.50$$

$$\therefore x = ₹\,\frac{2337.50 \times 2000}{187} = ₹\;25{,}000$$

His savings $= ₹\,25{,}000$, His investment in company:

$$A = \frac{20}{100} \times = ₹\,25{,}000 = ₹\,5{,}000,\quad B = \frac{30}{100} \times = ₹\,25{,}000 = ₹\,7{,}500,$$

$$C = \frac{25}{100} \times = ₹\,25{,}000 = ₹\,6{,}250$$

Example 5 .

Mr. Edwin Flynn buys ₹ 40 shares in a company which pays 12% dividend. He buys the shares at such a price that he gets 16% per annum on his investment. At what price did he buy each share?

Solution:

Face or nominal value of each share = ₹ 40

Annual income from each share = 12% of ₹ 40 = ₹ $\left(40 \times \frac{12}{100}\right)$ = ₹ 4.80

Suppose, Edwin buys each share for ₹ x

Then, his investment in each share = ₹ x

$\therefore$ Annual income from each share = 16% of ₹ x = ₹ $\left(x \times \frac{16}{100}\right)$ = ₹ $\frac{4x}{25}$

$$\therefore \frac{4x}{25} = 4.80$$

$$\Rightarrow x = \frac{4.80 \times 25}{4} = 30$$

Therefore Edwin Flynn buys each share for ₹ 30. **Ans.**

Linear Inequations

[CHAPTER – 4]

Question 1: Solve the following inequation and represent the solution set on the number line:

$$4x-19<\frac{3x}{5}-2\le\frac{-2}{5}+x,\ x\in R$$

Answer: $4x-19<\frac{3x}{5}-2\le\frac{-2}{5}+x$

Now $4x-19<\frac{3x}{5}-2$; $\frac{3x}{5}-2\le\frac{-2}{5}+x$

$\Rightarrow\quad 4x-19<\frac{3x-10}{5}$; $\frac{3x-10}{5}\le\frac{-2+5x}{5}$

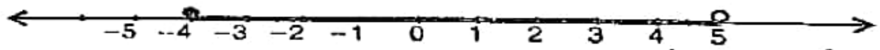
$\Rightarrow\quad 20x-95<3x-10$; $3x-10\le -2+5x$

$\Rightarrow\quad 17x<85$; $-2x\le 8$

$\Rightarrow\quad x<5$; $-x\le 4$ or $x\ge -4$

Hence, $-4\le x\le 5$

Number line is



Question 2: Solve the following inequation and represent the solution set on the number line:

$$2x-5\le 5x+4<11,\ x\in I$$

Answer: Given, inequation is $2x-5\le 5x+4<11,\ x\in I$

We have $2x-5<5x+4$; $5x+4<11$

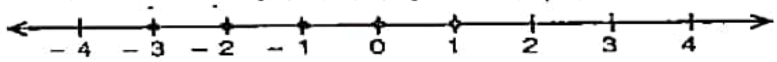
or $-5-4<5x-2x$; $5x<11-4$

$-9<3x$; $5x<7$

$-3<x$; $x<\frac{7}{5}$

$-3<x$; $x<1.4$

The solution set is [-3, -2,-1, 0, 1]and the representation on the number line is



Question 3: Solve the following inequation and represent the solution set on the number line:

$$-3<-\frac{1}{2}-\frac{2x}{3}\le\frac{5}{6},\ x\in R$$

Answer: Given, inequation is $-3<-\frac{1}{2}-\frac{2x}{3}\le\frac{5}{6},\ x\in R$

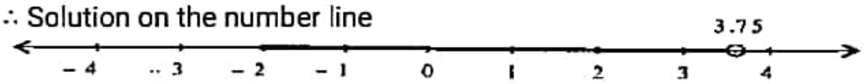
$\Rightarrow\quad -3<\frac{-3-4x}{6}\le\frac{5}{6}\Rightarrow -18<-3-4x\le 5$

$\Rightarrow\quad -18+3<-4x\le 5+3\ \Rightarrow -15<-4x\le 8$

$\Rightarrow\quad \frac{-15}{-4}>x\ge\frac{8}{-4}$

$\Rightarrow\quad 3.75>x\ge -2\qquad\Rightarrow -2\le x<3.75$

$\therefore$ Solution on the number line



Question 4: Solve the following inequation and graph the solution set on the number line:

$$-2\frac{2}{3}\le x+\frac{1}{3}<3\frac{1}{3};\ x\in R$$

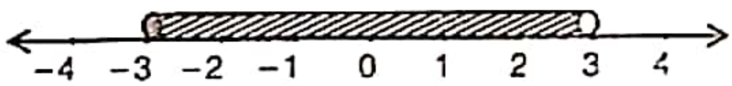
Answer: $-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}; x \in R$

$\Longrightarrow \frac{-8}{3} \le x + \frac{1}{3} < \frac{10}{3}$(By subtracting $\frac{1}{3}$ we get)

$\Longrightarrow \frac{-8}{3} - \frac{1}{3} \le x + \frac{1}{3} - \frac{1}{3} < \frac{10}{3} - \frac{1}{3} \Longrightarrow \frac{-9}{3} \le x < \frac{9}{3} \Longrightarrow -3 \le x < 3$

Solution is { $x : x \in R, -3 \le x < 3$}

On the number line solution is :



Question 5: Solve the following inequation and graph the solution set on the number line:

$2y - 3 \le y + 1 \le 4y + 7 ; y \in R$

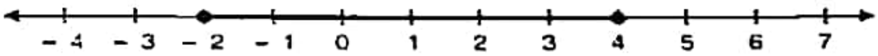
Answer: $2y - 3 \le y + 1 \le 4y + 7 ; y \in R$

or $2y - 3 - 1 \le y \le 4y + 7 - 1$

or $2y - 4 \le y$ and $y \le 4y + 6$

$\Longrightarrow$ $y \le 4$ and $-6 \le 3y$

$\therefore$ $-2 \le y \le 4$



Question 6: Solve the following inequation $x + \frac{2}{15} \le \frac{-8}{15}, x \in R$

Answer: $x + \frac{2}{15} \le \frac{-8}{15}, x \in R$

$\Longrightarrow 15x + 2 \le -8$ (multiply by 15), no change in sign inequality.

$\Longrightarrow 15x \le -10$ (add-2), no change in sign of inequality.

$\Longrightarrow x \le \frac{-2}{3}, x \in R$

Question 7: Solve the following inequation $-2\frac{3}{4} \le x + \frac{1}{4} < 4\frac{1}{4}, x \in R$

Answer: $-2\frac{3}{4} \le x + \frac{1}{4} < 4\frac{1}{4}, x \in R$

$\Longrightarrow \frac{-11}{4} \le \frac{4x+1}{4} < \frac{17}{4}$(multiply by 4)

$\Longrightarrow -11 \le 4x + 1 < 17$ $\Longrightarrow -11 - 1 \le 4x < 17 - 1$

$\Longrightarrow -12 \le 4x < 16$ $\Longrightarrow -3 \le x < 4$

Question 8: Solve the following inequation $\frac{1}{2}\left(\frac{3x}{2} + 4\right) \ge \frac{1}{3}(x - 6), x \in R$

Answer: $\frac{1}{2}\left(\frac{3x}{2} + 4\right) \ge \frac{1}{3}(x - 6), x \in R$

$\Longrightarrow$ $\frac{1}{2}\left(\frac{3x+8}{2}\right) \ge \frac{1}{3}(x - 6)$ $\Longrightarrow \frac{3x}{4} + 2 \ge \frac{x}{3} - 2$

$\Longrightarrow$ $\frac{3x+8}{4} \ge \frac{x-6}{3}$ (muliply by 12)

$\Longrightarrow$ $9x + 24 \ge 4x - 24$, now solve it

$\Longrightarrow$ $9x - 4x \ge 24 - 24$

$\Longrightarrow$ $5x \ge -48 \Longrightarrow x \ge \frac{-48}{5}$

Question 9: Solve the following inequation $\frac{2x+1}{2} + 2(3-x) \geq 7,\ x \in R$

Answer: $\frac{2x+1}{2} + 2(3-x) \geq 7,\ x \in R$

$\Longrightarrow \quad \frac{12x+1+12-4x}{2} \geq 7 \Longrightarrow -2x + 13 > 14$

$\Longrightarrow \quad -2x \geq 14-13$ [subtract 13]

$\Longrightarrow \quad -2x \geq 1$ [Divide by-2]

$\Longrightarrow \quad x \leq -\frac{1}{2}$

Question 10: x and y are I^+, such that $2 \leq x \leq 5$ and $3x + 2y \leq 12$. List the possible values of y.

Answer: $x = 2, 3, 4, 5$ and $3x + 2y \leq 12$

Put $x = 2$, So, $3\times2 + 2y \leq 12$

$\Longrightarrow \quad 2y \leq 6$

$\Longrightarrow \quad y \leq 3$

So, $y = \{1,2,3\}$

Quadratic equations

[CHAPTER – 5]

Question 1: Solve the following equation and give your answer correct to 3 significant figures :

$5x^2 - 3x - 4 = 0$

Answer: $5x^2 - 3x - 4 = 0$, Here, $a = 5,\ b = -3,\ c = 4$

From formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$\Longrightarrow \quad x = \frac{-(-3) \pm \sqrt{(-3)^2-(4\times5\times-4)}}{25}$

$\Longrightarrow \quad x = \frac{3 \pm \sqrt{9+80}}{10}$

$\Longrightarrow \quad x = \frac{3 \pm \sqrt{89}}{10}$

$\Longrightarrow \quad x = \frac{3 \pm 9.486}{10}$

$\Longrightarrow \quad x = \frac{3 + 9.486}{10}$, $x = \frac{3- 9.486}{10}$

$\Longrightarrow \quad x = \frac{12.486}{10}$, $\frac{- 6.486}{10}$

$\Longrightarrow \quad x = 1.249$, -0.649 = 1.25, -0.649 Correct to three sig fig.

Question 2: Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots. $x^2 + 2(m-1)x + (m+5) = 0$

Answer: $x^2 + 2(m-1)x + (m+5) = 0$, Here, $a = 1,\ b = 2(m-1),\ c = m + 5$

Discriminant, $D = b^2 - 4ac$

$= [2(m-1)]^2 - 4\times1(m + 5)$

$= 4(m^2-2m+1) - 4m - 20$

$= 4m^2 - 8m + 4 - 4m - 20$
$= 4m^2 - 12m - 16$
$= 4(m^2 - 3m - 4)$

For real and equal roots $D = 0$

Hence, $4(m^2 - 3m - 4) = 0$

$\Longrightarrow 4 \neq 0 \quad m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

So, $m = -1, 4$

Question 3: Solve the following equation:

$x - \frac{18}{x} = 6$. Give your correct answer to 2 significant figures.

Answer: Given, quadratic equation is $x - \frac{18}{x} = 6$

$\Longrightarrow \frac{x^2 - 18}{x} = 6 \qquad \Longrightarrow x^2 - 18 = 6x$

$\Longrightarrow x^2 - 6x - 18 = 0$

We have, Discriminant (D) $= b^2 - 4ac$

$= (-6)^2 - 4\times1\ (-18) = 36 + 72 = 108$
$= 36 + 72 = 108$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{108}}{2\times1}$$
$$= \frac{(6) \pm 6\sqrt{3}}{2} = \frac{2(3 \pm 2\sqrt{3})}{2}$$
$$= 3 \pm 2\sqrt{3} = 3 \pm 2\times1.732 = 3 \pm 3.464$$

$\Longrightarrow \quad x = 3 + 3.464$ and $x = 3 - 3.464$

$\Longrightarrow \quad x = 6.464$ and $x = -0.464$

$\therefore \quad x = \{-0.46, 6.5\}$ Correct to two sig fig.

Question 4: Solve the following equation and give your answer correct to two decimal places:

$5x(x + 2) = 3$

Answer: $5x(x + 2) = 3 \quad \Longrightarrow 5x^2 - 10x - 3 = 0$, Here, $a = 5$, $b = 10$, $c = -3$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} == \frac{-10 \pm \sqrt{(-10)^2 - (4\times5\times-3)}}{2\times5}$

$\Longrightarrow \quad x = \frac{-10 \pm \sqrt{100+60}}{10}$

$\Longrightarrow \quad x = \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 12.65}{10}$

$\Longrightarrow \quad x = \frac{-10 + 12.65}{10}, x = \frac{-10 - 12.65}{10}$

$\Longrightarrow \quad x = \frac{2.65}{10}, \frac{-22.65}{10}$

$\Longrightarrow \quad x = 0.265, -2.265 = 0.27, -2.27$

Question 5: Solve the following equation for x and give your and give your answer correct to two decimal places: x^2-3x-9=0

Answer: $x^2 - 3x - 9 = 0$, Here, $a = 1$, $b = -3$, $c = -9$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2-(4\times1\times-9)}}{2\times1}$

$\Longrightarrow \quad x = \frac{3 \pm \sqrt{9+36}}{2}$

$\Longrightarrow \quad x = \frac{3\pm \sqrt{45}}{2} = \frac{3 \pm 6.71}{2}$

$\Longrightarrow \quad x = \frac{3 + 6.708}{10} \quad , x = \frac{3- 6.708}{2}$

$\Longrightarrow \quad x = \frac{9.708}{2} \quad , \frac{-3.708}{2}$

$\Longrightarrow \quad x = 4.854 \quad ,-1.85\,4$

$\therefore \quad X = 4.85, - 1.85$ Correct to two dcp.

Question 6: Solve the following equation by using formula only and give the answer correct to two decimal places 2(x − 1) (x − 5) = 5

Answer: $2(x-1)(x-5) = 5$

$\Longrightarrow 2(x^2-6x+5) = 5 \quad \Longrightarrow 2x^2-12x + 5 = 0$

So, $\quad x = \frac{-(-12) \pm \sqrt{(-12)^2-(4\times2\times5)}}{2\times2} = \frac{12 \pm \sqrt{104}}{4}$

$\Longrightarrow \quad x = \frac{12 \pm 10.19}{4} \Longrightarrow x = \frac{12 + 10.19}{4}, \quad x = \frac{12- 10.19}{4}$

$\Longrightarrow \quad x = \frac{22.19}{4} \quad ,\frac{1.81}{4} \Longrightarrow x = 5.55 \quad , \ 0.45$

Question 7: Solve the following equation: x (x+1) + (x+2) (x+3) = 42

Answer: $x (x + 1) + (x + 2) (x + 3) = 42$

$\Longrightarrow \quad 2x^2 + 6x-36 = 0 \quad \Longrightarrow \quad x^2 + 3x-18 = 0$

$\Longrightarrow \quad (x+6)(x+3) = 0 \quad \Longrightarrow \quad x = -6 , \quad x = 3$

Question 8: Solve for x : $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$

Answer: $\frac{x+3}{x+2} = \frac{3x-7}{2x-3}$, By cross-multiplication, we get

$\Longrightarrow \quad 2x^2 + 6x-3x-9 = 3x^2 + 6x-7x-14$

$\Longrightarrow \quad x^2-4x-5 = 0 \quad \Longrightarrow \quad x^2-5x + x-5 = 0$

$\Longrightarrow \quad x(x-5) + 1(x-5) = 0 \quad \Longrightarrow \quad (x-5)(x+1) = 0$

$\Longrightarrow \quad x = 5, -1$

Question 9: Solve for x : $2x-\frac{3}{x} = 5$

Answer: $2x- \frac{3}{x} = 5 \quad \Longrightarrow 2x^2-5x-3 = 0$

$\Longrightarrow \quad 2x^2-6x + x-3 = 0$

$\Longrightarrow \quad 2x(x-3) + 1(x-3) = 0$

$\Longrightarrow \quad 2x + 1 = 0 \quad$ or $\quad x-3 = 0$

$\Longrightarrow \quad x = \frac{-1}{2},$ or 3

Question 10: Solve the given quadratic equation by using formula and give the answer correct to two decimal places: $5(x+1)^2+10(x+1)+3=0$

Answer: $5(x+1)^2 + 10(x+1) + 3 = 0$

Let $x + 1 = a$

$\therefore 5a^2 + 10a + 3 = 0$

By formula, $a = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-10 \pm \sqrt{(10)^2-(4\times5\times3)}}{2\times5}$

$\Rightarrow \quad = \dfrac{-10 \pm \sqrt{100-60}}{10}$

$\Rightarrow \quad = \dfrac{-10 \pm \sqrt{40}}{10} = \dfrac{-10 \pm 6.32}{10}$

$\Rightarrow \quad = \dfrac{-10 + 6.32}{10}, x = \dfrac{-10 - 6.32}{10}$

$\Rightarrow \quad a = \dfrac{-3.68}{10}, \dfrac{-16.32}{10}$

$\Rightarrow \quad x = -0.368, -1.632$

Since $a = x + 1$

$\Rightarrow \quad x + 1 = -0.368$ or $x + 1 = -1.632$

$= -1.368 \qquad = -2.632$

$= -1.37 \qquad = -2.63$

$\Rightarrow \quad x = -1.37, -2.63$

Question 11: Solve for x : $\dfrac{x+1}{x-2} + \dfrac{x+11}{x+3} = 4$

Answer: $\dfrac{x+1}{x-2} + \dfrac{x+11}{x+3} = 4$

$\Rightarrow \dfrac{x^2+4x+3+x^2+9x-22}{(x-2)(x+3)} = 4$

$\Rightarrow \quad 2x^2 + 13x - 9 = 4(x-2)(x+3)$

$\Rightarrow \quad 2x^2 + 13x - 19 - 4x^2 - 4x - 24 = 0$

$\Rightarrow \quad -2x^2 + 9x + 5 = 0$

$\Rightarrow \quad 2x^2 - 9x - 5 = 0$

$\Rightarrow \quad 2x(x-5) + 1(x-5) = 0$

$\Rightarrow \quad (2x+1)(x-5) = 0$

$\Rightarrow x = -\dfrac{1}{2}, 5$

Problems based on Quadratic equations [CHAPTER – 6]

Q.1. Three consecutive positive odd numbers are such that the sum of the squares of the first two numbers greater than the square of the third by 65. Find the numbers.

Ans. Let the three consecutive odd numbers be $(2x-1),(2x+1),(2x+3)$ Then,

$$(2x-1)^2+(2x+1)^2=(2x+3)^2+65$$

$$\Rightarrow 4x^2-4x+1+4x^2+4x+1=4x^2+12x+9+65$$

$$\Rightarrow 4x^2-12x-72=0$$

$$\Rightarrow x^2-3x-18=0 \Rightarrow (x-6)(x+3)=0$$

$$\Rightarrow x=6 \; and \; x=-3$$

More $x=-3$ is an inadmissible solution as odd numbers are non-negative the three numbers are 11,13 and 15.

Q.2. A two-digit number is such that the products of its digit are 12. When 36 is added to the number, the digits are inter changed. Find the number.

Ans. Let the ones digit of the two digit number x since the product of its digit is 12, the ten digit is $\frac{12}{x}$

$\therefore$ The two digit number is $10\left(\frac{12}{x}\right)+x$

On inter changing the digits, the number

Becomes $10x+\frac{12}{x}$

According to the question, $10\left(\frac{12}{x}\right)+x+36=10x+\frac{12}{x}$

$$\Rightarrow \frac{120}{x}+x+36=10x+\frac{12}{x}$$

$$\Rightarrow 120+x^2+36x=10x^2+12$$

$$\Rightarrow 9x^2-36x-108=0$$

$$\Rightarrow x^2-4x-12=0$$

$$\Rightarrow (x-6)(x+2)=0$$

$\Rightarrow x-6=0 \ or \ x+2=0$

$\Rightarrow x=6 \ or -2$

Here x cannot be -2, as the digits of a number cannot be negative.

So, $x=6 \Rightarrow$ ones digit is 6 and the tens digit is 12, i.e. 2.

More the required number is 26.

Q.3. The perimeter of a right angle of triangle is five times the length of its shortest side. The numerical value of the area of the triangle is 15 times the numerical value of the length of the shortest side. Find the lengths of the three sides of the triangle.

Ans. Let $x, y, z(x>y>z)$ be the lengths of the three sides of the right angled triangle, Then $x+y+z=52$ ... (1)

And Area of the triangle = $\frac{y2}{2}=152$ ....(2)

Also, $x^2 = y^2 + z^2$ (3)

From (1) and (2), we get y=30 and x=4z-30

$\Rightarrow$using these values in (3), we get z=16

$\Rightarrow x=4(16)-30=64-30=34$

Hence, the three sides of the right angled triangle are 34cm, 30cm and 16cm.

Q.4. a triangle of 105sq.cm has its length as x cm. Write down its breadth in terms of x. Given, that its perimeter is 44cm. Write down an equation in x and solve it to determine the dimensions of the rectangle.

Ans. We know that for a rectangle

$$Breadth=\frac{Area}{length} \Rightarrow breadth=\frac{105cm}{x}$$

Perimeter of the rectangle = 2(ℓ l+b)cm

So, perimeter $= 2\left[x+\frac{105}{x}\right]cm = 2\frac{(x^2+105)}{x}cm$

Since perimeter is given as 44 cm, we have

$2(x^2+105) = 44x$

$\Rightarrow x^2 - 22x + 105 = 0$

So, using the equation for x, we have

$x = 15 \ or \ x = 7$

Thus, the length and breadth of the rectangle are 15cm and 7 cm.

Q.5. A rectangular garden 10m by 16m is to be surrounded by a concrete path of uniform width. Given that area of the path is 120sqm and as using the width of the path to be x, form an equation in x and solve it to find the value of x.

Ans. Area of the garden without path = 160m^2

Area of garden with path $= (10+2x)(16+2x)\,m^2$

According to the question, $(10+2x)(16+2x) - 160 = 120$

$\Rightarrow x^2 + 13x - 30 = 0 \Rightarrow (x-2)(x+15) = 0$

Here, $x = 15$ is an inadmissible value. So, $x = 2m$.

Q.6. P and Q are the centres of two circles with radii 9cm and 2cm. PQ=17CM, R is the centre of a circle of radius $x\ cm$, which touches the above circles eternally.

Given that $\angle PRQ = 90°$. Write an equation in x and solve it for x.

Ans. By Pythagoras theorem, we have

$\Rightarrow (17)^2 = (9+x)^2 + (2+x)^2$

$\Rightarrow 289 = 81 + 18x + x^2 + 4 + 4x + x^2$

$\Rightarrow 2x^2 + 22x - 104 = 0$

$\Rightarrow x^2 + 11x - 102 = 0$

$\Rightarrow (x-6)(x+17) = 0$

$\Rightarrow x - 6 = 0 \text{ or } x + 17 = 0$

$\Rightarrow x = 6 \text{ or } x = -17$

But, $x = -17$ is an inadmissible value,

Hence, x = 6.

Q.7. Two years ago, Jacob's age was the three times the square of John's age. In three years, time John's age will be one-fourth of Jacob's age. Find their present ages.

Ans. Let John's present age (in years) be x. Then his age two years ago was (x-2)

$\Rightarrow$ Jacob'sage, 2yearsago, was $3(x-2)^2$

$\Rightarrow Jacob' spresent\, age = 3(x-2)^2 + 2$

After 3 years, age of John = $(x+3)$

Age of Jacob = $3(x-2)^2 + 2 + 3$

$= 3(x-2)^2 + 5$

According to the equation, $3(x-2)^2 + 5 = 4(x+3)$

$\Rightarrow 3(x^2 - 4x + 4) + 5 = 4x + 12$

$\Rightarrow 3x^2 - 12x + 12 + 5 = 4x + 12$

$\Rightarrow 3x^2 - 16x + 5 = 0$

$\Rightarrow 3x^2 - 15x - x + 5 = 0$

$\Rightarrow 3x(x-5) - 1(x-5) = 0$

$\Rightarrow (3x-1)(x-5) = 0$

$\Rightarrow 3x-1=0 \; or \; x-5=0$

$\Rightarrow x=\frac{1}{3} \; or \; x=5$

Rejecting the value of x as $\frac{1}{3}$, we have x=5.

Thus their present ages are 5 years and 29 years.

Q.8. By increasing the speed of a car by 10km/hr, the time of the journey for a distance of 72 km. is reduced by 36 minutes. Find the original speed of the car.

Ans. Let the original speed of the car be $x\,km/hr$ Then, the increased speed= $(x+10)km/hr$ Time taken with the speed of $x\,km/hr=\frac{72}{x}$ hours.

Time taken with the speed of $(x+10)km/hr=\frac{72}{x+10}$ hours.

According to the question, $\frac{72}{x}-\frac{72}{x+10}=\frac{36}{60}$

$\Rightarrow 72[(x+10)-x]=\frac{36}{60}x(x+10)$

$\Rightarrow 72(x+(0-x)=\frac{36x(x+10)}{60}$

$\Rightarrow 72\times 10=\frac{3x(x+10)}{5}$

$\Rightarrow 20\times 60=x(x+10)$

$\Rightarrow x^2+10x-1200=0$

$\Rightarrow x^2+40x-30x-1200=0$

$\Rightarrow (x+40)(x-30)=0$

$\Rightarrow x+40=0 \; or \; x-30=0$

$\Rightarrow x=-40 \; or \; 30$

Since speed cannot be negative, $x=-40$ is an in admissible value.

Hence, the original speed of the car is 30km/hr.

Q.9. Swati can row her boat at a speed of 5km/hr in still water. If it takes 1 hours more to row the boat 5.25km upstream than to return back downstream, find the speed of the stream.

Ans. Let the speed of the stream be x km/hr.

Speed of the boat upstream= $(5-x)km/hr$

Speed of the boat downstream= $(5+x)\,km/hr$

According to the question, $\frac{5.25}{5-x}-\frac{5.25}{5+x}=1$

$\Rightarrow 2x^2+21x-50=0$

$\Rightarrow 2x^2+25x-4x-50=0$

$\Rightarrow (x-2)(2x+25)=0$

$\Rightarrow x=2\ or\ \ x=-\frac{25}{2}$ (In admissible)

Since speed can not be negative, $x=-\frac{25}{2}$ is rejected

Thus, the speed of the stream is 2km/hr.

Q.10. A takes 10 days less than time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

Ans. Suppose B alone takes x days to finish the work and A alone can finish it in $(x-10)$ days. The (A' one day work)+(B's one days work)

$\frac{1}{x}+\frac{1}{x-10}$

(A+B)'s one day work $= \frac{1}{12}$

$\therefore \frac{1}{x}+\frac{1}{x-10}=\frac{1}{12} \Rightarrow x^2-34x+120=0$

$\Rightarrow x^2-30x-4x+720=0$

$\Rightarrow (x-30)(x-4)=0$

$\Rightarrow x=30 \text{ or } x=4$

Since x can not be less than 10 the value of x is 30

Thus B alone can finish the work in 30 days.

Q.11. A shopkeeper buys a certain number of books for Rs.720. If the cost per books was Rs.5 less, the number of books that could be bought for Rs.720 would be 2 more. Taking the original cost of each book to be Rs.x, Write down an equation in x and solve.

Ans. Let the cost per book be Rs.x. Then number of books that can be bought with Rs.2700$=\frac{720}{x}$

According to the question,

$\frac{720}{x-5}=\frac{720}{x}+2$

$\Rightarrow \frac{720}{x-5}-\frac{720}{x}=2$

$\Rightarrow 720\left(\frac{1}{x-5}-\frac{1}{x}\right)=2 \Rightarrow \frac{120\times5}{x(x-5)}=2$

$\Rightarrow x(x-5)-1800=0 \Rightarrow x^2-5x-1800=0$

$\Rightarrow x^2-45x+40x-1800=0$

$\Rightarrow x(x-45)+40(x-45)=0$

$\Rightarrow (x-45)(x+40)=0$

$\Rightarrow x = 45\ or\ x = -40$

Since cost can not be negative, so x=40 is an in admissible value.

Hence, x=45

Q.12. Vinayak bought a calculation for Rs.60x and sold it for Rs(600-6x) at a loss of x%. Find its selling price.

Ans. Here,

C.P=Rs.60x and S.P.= Rs.(600-6x)

Loss = C.P.-S.P.

= Rs.(60x-600+6x)

=Rs(66x-600)

Loss%= $\frac{66x - 600}{60x} \times 100$

Equating it to x%, we have $\left(\frac{66x - 600}{60x}\right) \times 100 = x$

$\Rightarrow 6,600x - 60,000 = 60x^2$

$\Rightarrow x^2 - 110x + 1000 = 0$

$\Rightarrow x^2 - 100x - 10x + 1000 = 0$

$\Rightarrow (x-10)(x-100) = 0$

$\Rightarrow x - 10 - 0\ or\ x - 100 = o$

$\Rightarrow x = 10\ or\ x = 100$

If $x = 100$, S.P. will be zero

So, $x = 100$ is an inadmissible value.

Thus, x=10 and S.P. = Rs(600- 6X10)

=Rs.540

Q.13. Find two natural numbers which differ by 3 and the sum of whose squares is 117.

Ans. Let the natural numbers. Be x and x+3.

$$\therefore (x^2 + (x+3)^2 = 117$$

$$\Rightarrow x^2 + x^2 + 6x + 9 = 117$$

$$\Rightarrow 2x^2 + 6x - 108 = 0$$

$$\Rightarrow x^2 + 3x - 54 = 0$$

$$\Rightarrow (x+9)(x-6) = 0$$

$$\Rightarrow x+9 = 0, \, or \; x-6 = 0$$

$$x = -9, \, or \; x = 6$$

$\therefore$ One number=6[since-9 is not a natural number and other number.

$$= 6+3 = 9$$

$\therefore$ Numbers are 6 and 9.

Q.14. Five times a certain whole number is equal to three less than twice the square of the number. Find the number.

Ans. Let the number be x.

Given, five times the number =3 less than twice the square of the number.

$$\therefore 5x = 2x^2 - 3$$

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$\Rightarrow 2x^2 - bx + x - 3 = 0$$

$$\Rightarrow (x-3)(2x+1) = 0$$

$$\Rightarrow (x-3) = 0, \, or \; (2x+1) = 0$$

$\Rightarrow x = 3 \ or \ x\frac{1}{2}$

$\therefore$ Required whole number is 3

Q.15. Sum of two natural numbers is 8 and the difference of their reciprocal is $\frac{2}{15}$. Find the numbers.

Ans. Let the natural numbers be x and 8-x

$\Rightarrow \frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$ i.e. $\frac{8-x-x}{x(8-x)} = \frac{2}{15}$

$\Rightarrow 2(8x - x^2) = 15(8 - 2x) i.e. 16x - 2x^2 = 120 - 30x$

$\Rightarrow 2x^2 - 46x + 120 = 0$ i.e. $x^2 - 23x + 60 = 0$

$\Rightarrow x^2 - 20x - 3x + 60 = 0$ i.e. $x(x-20) - 3(x-20) = 0$

$\Rightarrow (x-20)(x-3) = 0$ i.e. $x - 20 = 0 \ or \ x - 3 = 0$

$\Rightarrow x = 20 \ or \ x = 3$

Reject x=20 as the sum of two natural numbers is 8

$\therefore x = 3 \ and \ 8 - x = 8 - 3 = 5$

$\therefore$ Required Natural numbers are 3 and 5.

Q.16. For the same amount of work, A takes 6 hours less than B. If together they complete the work in 13 hours and 20 minutes. Find how much time will B alone take to complete the work?

Ans. If B alone takes x hours and A alone takes (x-6) hours for the same work.

$\frac{1}{x-6} + \frac{1}{x} = \frac{3}{40} \left[\because 13 hrs 20 \min = 13 + \frac{20}{60} = \frac{40}{3} hrs \right]$

$\Rightarrow \frac{x + x - 6}{(x-6)x} = \frac{3}{40}$

$\Rightarrow 3x^2 - 18x = 80x - 240$ i.e. $3x^2 - 98x + 240 = 0$

$\Rightarrow 3x^2 - 90x - 8x + 240 = 0$ *i.e.* $(x-30)(3x-8) = 0$

$\Rightarrow x = 30, \text{ or } x = \frac{8}{3}$ *i.e.* $x = 30$

$\therefore$ B alone will take 30 hrs to complete the work.

Q.17. The hypotenuse of a right triangle is 13 cm and the difference between the other two sides is 7cm.

Taking 'x' as the length of the shorter of the two sides, write an equation in 'x' that represents the above statement and also solve the equation to find the two unknown sides of triangle.

Ans. Since, the shorter side $= x cm$

$\therefore$ longer side $== (x+7)\,cm$

Using Pythagoras Theorem, we get:

$x^2 + (x+7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 + 14x = 169$

$\Rightarrow 2x^2 + 14x - 120 = 0$

$\Rightarrow x^2 + 7x - 60 = 0$

On solving, it gives $x = -12, \text{ or } x = 5$

Since, the side of triangle can not be negative, therefore, $x = 5$

$\therefore$ One side of the triangle = x cm = 5 cm and other side of the triangle $= (x+7)\,cm = (5+7)\,cm = 12\,cm$

Q.18. Car A travels $x\,km$ for every litre of petrol, while car B travels $(x+5)\,km$ for every litre of petrol.

i) Write down the number of litres of petrol used by car A and Car B in covering a distance of 400km.

ii) If Car A uses 4 litres of petrol more than Car B in covering the 400 km, write down an equation in x and solve to determine the number of litres of petrol used by Car B for the journey.

Ans. i) No. of lit5es of petrol used by Car A= $\frac{400}{x}$ *litre*

No. of litres of petrol used by Car B = $\frac{400}{x+5}$ *litre*

ii) Given : $\frac{400}{x} - \frac{400}{x+5} = 4$ *i.e.* $\frac{400x+2000-400x}{x(x+5)} = 4$

$\Rightarrow 4(x^2+5x) = 2000$ *i.e.* $x^2+5x-500=0$

$\Rightarrow x^2+25x-20x-500=0$

$\Rightarrow x = 25,$ *or* $x = -20$

$\Rightarrow x = 20$($\because$ Distance cannot be negative)

$\therefore$ No. of litres of petrol used by Car B= $\frac{400}{x+5}$ *litres*= $\frac{400}{20+5}$ =16*litres.*

Q.19. By selling an article for Rs.24, a trader loses as much percent as the cost price of the article. Calculate the cost price.

Ans. Let C.P. of the article be Rs.x

$\therefore$ *loss* = *x*% *of C.P* = $\frac{x}{100} \times Rs.x = \frac{Rs.x^2}{100}$

$\therefore x - \frac{x^2}{100} = 24$ [C.P. – loss =S.P.]

$\Rightarrow 100x - x^2 = 2400$ *i.e.* $x^2 - 100x + 2400 = 0$

$\Rightarrow x^2 - 60x - 40x + 2400 = 0$

On solving, we get: x=60 and x=40

$\therefore$ C.P. of the article is Rs.60 or Rs.40.

Q.20. The sum S of first n natural numbers is given by the relation $s=\frac{1}{2}n(n+1)$

Find n, if the sum is 276.

Given : S=276

$\Rightarrow \frac{1}{2}n(n+1)=276 \text{ i.e. } n^2+n-552=0$

$=n^2+24n-23n-552=0$

$\Rightarrow (n+24)(n-23)=0$

$\Rightarrow n=-24, \text{ or } n=23$ [Zero product rule]

Since, n is a natural number, reject n=-24. $\therefore$ n=23.

Matrix

[CHAPTER – 9]

Q 1. Find x, y, z and a for which $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$

Sol. $x+3=0$, $\quad 2y+x=-7$

$z-1=3$, $\quad 4a-6=2a$

So, $x+3=0$

$\Rightarrow x=-3$

$2y+x=-7$

$\Rightarrow 2y+(-3)=-7$

$\Rightarrow 2y=-4$

$\Rightarrow y=-2$

$z-1=3$

$\Rightarrow z=4$

$4a-6=2a$

$\Rightarrow 2a=6$

$\Rightarrow a=3$

Q 2. If A = $\begin{bmatrix} 1 & 2 \\ -1 & 6 \end{bmatrix}$**, find its transpose matrix.**

Sol. If A = $\begin{bmatrix} 1 & 2 \\ -1 & 6 \end{bmatrix}$, then A′ = $\begin{bmatrix} 1 & -1 \\ 2 & 6 \end{bmatrix}$

Ex.4. If A = $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$**, find A + A′.**

Sol. If A = $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, then A′ = $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

$\therefore \quad A + A' = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$

Q 3. If A = diag [3 5] and B = diag [2 – 3], find

(i) A + 2B **(ii) 3B – B**

Sol. Given $A = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$

(i) $A + 2B = \begin{bmatrix} 3+2\times2 & 0+2\times0 \\ 0+2\times0 & 5+2\times(-3) \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}$

(ii) $3B - B = \begin{bmatrix} 3\times2-2 & 3\times0-0 \\ 3\times0-0 & 3\times(-3)+3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -6 \end{bmatrix}$

Q 4. If $A = \begin{bmatrix} \sin^2\theta & -\cos^2\theta \\ 1 & \sec^2\theta \end{bmatrix}$ **and** $B = \begin{bmatrix} \cos^2\theta & -\sin^2\theta \\ 0 & -\tan^2\theta \end{bmatrix}$, **find A + B.**

Sol. $A + B = \begin{bmatrix} \sin^2\theta+\cos^2\theta & -\cos^2\theta-\sin^2\theta \\ 1+0 & \sec^2\theta-\tan^2\theta \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Q 5. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ **and** $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Sol. $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

$\Rightarrow$ $2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$

So, $X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

Q 6. Solve the equation: $\begin{bmatrix} 5 & 4 \\ 8 & -3 \end{bmatrix} - 3X = \begin{bmatrix} -4 & 7 \\ 5 & -9 \end{bmatrix}$

Sol. $\begin{bmatrix} 5 & 4 \\ 8 & -3 \end{bmatrix} - 3X = \begin{bmatrix} -4 & 7 \\ 5 & -9 \end{bmatrix}$

$\Rightarrow$ $3X = \begin{bmatrix} 5 & 4 \\ 8 & -3 \end{bmatrix} - \begin{bmatrix} -4 & 7 \\ 5 & -9 \end{bmatrix}$

$\Rightarrow$ $3X = \begin{bmatrix} 9 & -3 \\ 3 & 6 \end{bmatrix}$

$\Rightarrow$ $X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Q 7. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 3 & 7 \end{bmatrix}$, find AB.

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 1\times4+2\times3 & 1\times(-1)+2\times7 \\ 3\times4+4\times3 & 3\times(-1)+4\times7 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 24 & 25 \end{bmatrix}$

Q 8. Evaluate x and y, if $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

Sol. Given $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix} \Rightarrow \begin{bmatrix} 6x-2-8 \\ -2x+4+10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

On comparing corresponding elements, we get

$$6x - 2 - 8 = 8$$
$$\Rightarrow 6x = 8 + 2 + 8$$
$$\Rightarrow 6x = 18$$
$$\Rightarrow x = 3$$

and

$$-2x + 4 + 10 = 4y$$
$$\Rightarrow -2x - 4y = -14$$
$$\Rightarrow (-2)\times 3 - 4y = -14$$
$$\Rightarrow -4y = -14 + 6$$
$$\Rightarrow -4y = -8$$
$$\Rightarrow y = 2$$

Q 9. Find a 2 × 2 matrix B, such that $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$.

Sol. Given $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$, let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6a+5c & 6b+5d \\ 5a+6c & 5b+6d \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$

On comparing corresponding elements, we get

$$6a + 5c = 11$$
$$5a + 6c = 0$$
$$6b + 5d = 0$$
$$5b + 6d = 11$$

Multiplying (*i*) by 6, we get $36a + 30c = 66$

Multiplying (*ii*) by 5, we get $25a + 30c = 0$

$$\underline{-\quad\quad -\quad\quad -}$$
$$11a = 66 \Rightarrow a = 6$$

Now, $\quad 6a + 5c = 11$

$\Rightarrow \quad 6 \times 6 + 5c = 11$

$\Rightarrow \quad 5c = 11 - 36$

$\Rightarrow \quad 5c = -25 \Rightarrow c = -5$

Multiplying (*iii*) by 6, we get $\quad 36b + 30d = 0$

Multiplying (*iv*) by 5, we get $\quad 25b + 30d = 55$

$$\underline{- \quad - \quad -}$$

$$11b = -55 \Rightarrow b = -5$$

$\Rightarrow \quad 6 \times (-5) + 5d = 0$

$\Rightarrow \quad -30 + 5d = 0$

$$d = \frac{30}{5} \Rightarrow d = 6$$

$\therefore$ Matrix B $= \begin{bmatrix} 6 & -5 \\ -5 & 6 \end{bmatrix}$

Q 10. Let A = $\begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Is the product AB possible? Give reason. If yes, find AB.

Sol. Yes, the product AB is possible.

Reason : Order of matrix A is 2×2 and that of matrix B is 2×1.

The number of columns in A are same as the number of rows in B.

$\therefore$ The resulting matrix AB will have order 2×1

$$A_{2 \times 2} \times B_{2 \times 1} = AB_{2 \times 1}$$

Now, $\quad AB = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} 3 \times 2 + 5 \times 4 \\ 4 \times 2 + (-2) \times 4 \end{bmatrix} = \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \end{bmatrix}$$

Q 11. Evaluate: $\begin{bmatrix} 4 \sin 30° & 2 \cos 60° \\ \sin 90° & 2 \cos 0° \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Sol. Given

$$\begin{bmatrix} 4 \sin 30° & 2 \cos 60° \\ \sin 90° & 2 \cos 0° \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 4 \times \frac{1}{2} & 2 \times \frac{1}{2} \\ 1 & 2 \times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 4 + 1 \times 5 & 2 \times 5 + 1 \times 4 \\ 1 \times 4 + 2 \times 5 & 1 \times 5 + 2 \times 4 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Q 12. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find $A^2 - 5A + 7I$.

Sol. Given $\quad A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then $\quad A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 \times 3 + 1 \times (-1) & 3 \times 1 + 1 \times 2 \\ (-1) \times 3 + 2 \times (-1) & (-1) \times 1 + 2 \times 2 \end{bmatrix}$

$$= \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$\therefore \quad A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 + 5 + 0 & 3 - 10 + 7 \end{bmatrix} = \begin{bmatrix} 15 - 15 & 0 \\ 0 & 10 - 10 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q 13. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. Find $A^2 + AB + B^2$.

Sol. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$\Rightarrow \quad A^2 = A \times A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

Now, $$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2+0 & 3+0 \\ 4-1 & 6+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

and $$B^2 = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

Now, $$A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 1+2+1 & 0+3+6 \\ 4+3-2 & 1+6-3 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$

Q 14. If $f(x) = x^2 - 5x + 7I$ and $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, find $f(A)$.

Sol. $$f(A) = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2-5+7 & 2-5+0 \\ 2-5+0 & 2-5+7 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 4 \end{bmatrix}.$$

Q 15. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, then find the value of k so that $A^2 = 8A + kI$.

Sol. $$A^2 = 8A + kI$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Now, by comparing corresponding elements, we get $k = -7$.

Q 16. Find a 2×2 matrix X which satisfies the equation

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

Sol. Take $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} \Rightarrow 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} - \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$\Rightarrow \quad 2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

Hence, $X = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$

Trigonometrical Identities

[CHAPTER – 21]

Q 1. Prove the identity : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Sol.
$$\text{LHS} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - 1)}$$
$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\{2(1 - \sin^2\theta) - 1\}} = \tan\theta\,\frac{(1 - 2\sin^2\theta)}{(2 - 2\sin^2\theta - 1)}$$
$$= \tan\theta\,\frac{(1 - 2\sin^2\theta)}{(1 - 2\sin^2\theta)}$$
$$= \tan\theta = \text{RHS}$$

Q 2. Prove the identity: $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$

Sol.
$$\text{LHS} = \sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B$$
$$= \cos^2 B\,(\sin^2 A + \cos^2 A) + \sin^2 B\,(\cos^2 A + \sin^2 A)$$
$$= \cos^2 B + \sin^2 B$$
$$= 1 = \text{RHS}$$

Hence proved.

Q 3. Prove that $2\sec^2\theta - \sec^4\theta - 2\,\text{cosec}^2\theta + \text{cosec}^4\theta = \cot^4\theta - \tan^4\theta$.

Sol.
$$\text{RHS} = \cot^4\theta - \tan^4\theta = (\cot^2\theta)^2 - (\tan^2\theta)^2$$
$$= (\text{cosec}^2\theta - 1)^2 - (\sec^2\theta - 1)^2$$
$$= \text{cosec}^4\theta + 1 - 2\text{cosec}^2\theta - (\sec^4\theta + 1 - 2\sec^2\theta)$$
$$= \text{cosec}^4\theta + 1 - 2\text{cosec}^2\theta - \sec^4\theta - 1 + 2\sec^2\theta$$
$$= 2\sec^2\theta - \sec^4\theta - 2\text{cosec}^2\theta + \text{cosec}^4\theta$$
$$= \text{LHS}$$

Hence proved.

Q 4. Prove the identity: $\sqrt{\dfrac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$

Sol.
$$\text{LHS} = \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sqrt{\frac{(1 - \sin\theta)}{(1 + \sin\theta)} \times \frac{(1 - \sin\theta)}{(1 - \sin\theta)}}$$ [Rationalising the denominator]
$$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}} = \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}} = \frac{1 - \sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= \sec\theta - \tan\theta = \text{RHS}$$

Hence proved.

Q 5. Prove that $(\text{cosec}\,A - \sin A)(\sec A - \cos A)\sec^2 A = \tan A$.

Sol.
$$\text{LHS} = (\text{cosec}\,A - \sin A)(\sec A - \cos A)\sec^2 A$$
$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\cdot\frac{1}{\cos^2 A}$$
$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\cdot\frac{1}{\cos^2 A}$$
$$= \frac{\cos^2 A}{\sin A}\cdot\frac{\sin^2 A}{\cos A}\cdot\frac{1}{\cos^2 A} = \tan A = \text{RHS}$$

Q 6. **Prove that** $\dfrac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\dfrac{\sin^3 A-\cos^3 A}{\sin A-\cos A}=2$

Sol. Using $a^3+b^3=(a+b)(a^2-ab+b^2)$ and $a^3-b^3=(a-b)(a^2+ab+b^2)$, we have

$$\text{LHS}=\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A}+\frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}$$

$$=\frac{(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)}{(\sin A+\cos A)}+\frac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{(\sin A-\cos A)}$$

$$=1-\sin A\cos A+1+\sin A\cos A$$

$$=2=\text{RHS}$$

Hence proved.

Q 7. **Prove the identity:** $\dfrac{\tan\theta}{\dfrac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta}=1$

Sol.

$$\text{LHS}=\frac{\tan\theta}{\dfrac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta}=\frac{\tan\theta}{\sin\theta\left(\dfrac{\sin^2\theta}{\cos\theta}+\cos\theta\right)}$$

$$=\frac{\tan\theta}{\dfrac{\sin\theta(\sin^2\theta+\cos^2\theta)}{\cos\theta}}=\frac{\tan\theta}{\tan\theta\times 1}=1=\text{RHS}$$

Hence proved.

Q 8. **Prove the identity:** $\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}=\sec\theta\operatorname{cosec}\theta$

Sol.

$$\text{LHS}=\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}=\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}=\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}}$$

$$=\sqrt{\frac{1}{\cos^2\theta\sin^2\theta}}=\sqrt{\sec^2\theta\operatorname{cosec}^2\theta}$$

$$=\sec\theta\operatorname{cosec}\theta$$

$$=\text{RHS}$$

Hence proved.

Q 9. **Prove the identity:**

(*i*) $\dfrac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$ (*ii*) $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\dfrac{1-\tan\theta}{1-\cot\theta}\right)^2$

Sol. (*i*)

$$\text{LHS}=\frac{\cot A+\tan B}{\cot B+\tan A}=\frac{(1/\tan A)+\tan B}{(1/\tan B)+\tan A}$$

$$=\frac{(1+\tan A\tan B)}{\tan A}\times\frac{\tan B}{(1+\tan A\tan B)}$$

$$=\cot A\tan B=\text{RHS}$$

Hence proved.

(*ii*)

$$\text{LHS}=\frac{1+\tan^2\theta}{1+\cot^2\theta}=\frac{\sec^2\theta}{\operatorname{cosec}^2\theta}=\frac{\sin^2\theta}{\cos^2\theta}=\tan^2\theta$$

$$\text{RHS}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\left[\frac{1-(\sin\theta/\cos\theta)}{1-(\cos\theta/\sin\theta)}\right]^2=\left[\frac{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}\right]^2$$

$$=\left[\frac{-(\sin\theta-\cos\theta)}{\cos\theta}\times\frac{\sin\theta}{(\sin\theta-\cos\theta)}\right]^2=\frac{\sin^2\theta}{\cos^2\theta}=\tan^2\theta$$

$\therefore$ LHS = RHS

Hence Proved

Q 10. Prove the identity: $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

Sol.

$$\begin{aligned} \text{LHS} &= (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 \\ &= 1 + \tan^2 A \tan^2 B + 2 \tan A \tan B + \tan^2 A + \tan^2 B - 2 \tan A \tan B \\ &= 1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B = \sec^2 A + \tan^2 B + \tan^2 A \tan^2 B \\ &= \sec^2 A + \tan^2 B \sec^2 A = \sec^2 A (1 + \tan^2 B) = \sec^2 A \sec^2 B \\ &= \text{RHS} \end{aligned}$$

Hence proved.

Q 11. Prove the identity: $\cos^6 A + \sin^6 A = 1 - 3\cos^2 A \sin^2 A$

Sol.

$$\begin{aligned} \text{LHS} &= \cos^6 A + \sin^6 A \\ &= (\cos^2 A)^3 + (\sin^2 A)^3 \qquad \{a^3 + b^3 = (a+b)^3 - 3ab(a+b)\} \\ &= (\cos^2 A + \sin^2 A)^3 - 3\cos^2 A \sin^2 A (\cos^2 A + \sin^2 A) \\ &= (1)^3 - 3\cos^2 A \sin^2 A (1) = 1 - 3\cos^2 A \sin^2 A \\ &= \text{RHS} \end{aligned}$$

Hence proved.

Q 12. Prove the identity: $\dfrac{1}{\operatorname{cosec} A - \cot A} - \dfrac{1}{\sin A} = \dfrac{1}{\sin A} - \dfrac{1}{\operatorname{cosec} A + \cot A}$

Sol.

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

Grouping similar terms, we get

$$\Rightarrow \frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\Rightarrow \frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A} \qquad \{\because \operatorname{cosec}^2 A - \cot^2 A = 1\}$$

$$\Rightarrow 2\operatorname{cosec} A = 2\operatorname{cosec} A$$

which is true.

Hence proved.

Q 13. Prove the identity: $\dfrac{\sin^2 A}{\cos^2 A} + \dfrac{\cos^2 A}{\sin^2 A} = \dfrac{1}{\sin^2 A \cos^2 A} - 2$

Sol.

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A \cos^2 A} - 2$$

$$\Rightarrow \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} + 2 = \frac{1}{\sin^2 A \cos^2 A}$$

$$\Rightarrow \frac{(\sin^2 A)^2 + (\cos^2 A)^2 + 2\sin^2 A \cos^2 A}{\cos^2 A \sin^2 A} = \frac{1}{\sin^2 A \cos^2 A}$$

$$\Rightarrow \frac{(\sin^2 A + \cos^2 A)^2}{\sin^2 A \cos^2 A} = \frac{1}{\sin^2 A \cos^2 A}$$

$$\Rightarrow \frac{1}{\sin^2 A \cos^2 A} = \frac{1}{\sin^2 A \cos^2 A}$$

Hence proved

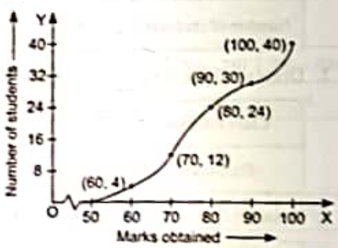
Graphical Representation

[CHAPTER – 23]

Q 1. Draw an Ogive for the following frequency distribution:

Marks obtained	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Number of students	4	8	12	6	10

Marks obtained	Number of students	Cumulative frequency
50 – 60	4	4
60 – 70	8	12
70 – 80	12	24
80 – 90	6	30
90 – 100	10	40

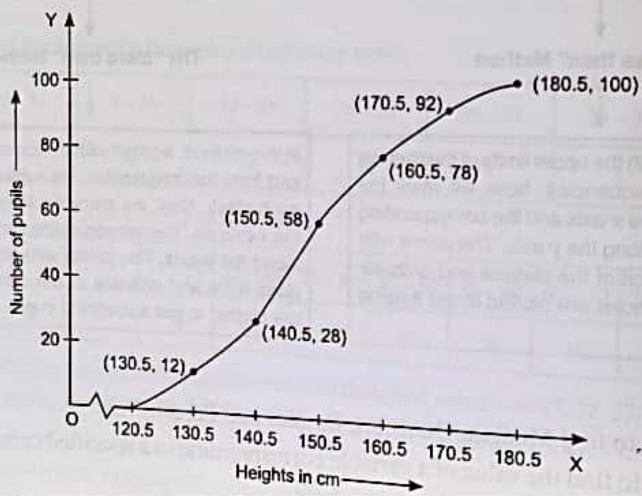


Q 2. Draw the Ogive for the following data:

Heights (in cm)	121 – 130	131 – 140	141 – 150	151 – 160	161 – 170	171 – 180
Number of pupils	12	16	30	20	14	8

Sol. Adjustment factor = $\frac{131-130}{2} = \frac{1}{2} = 0.5$

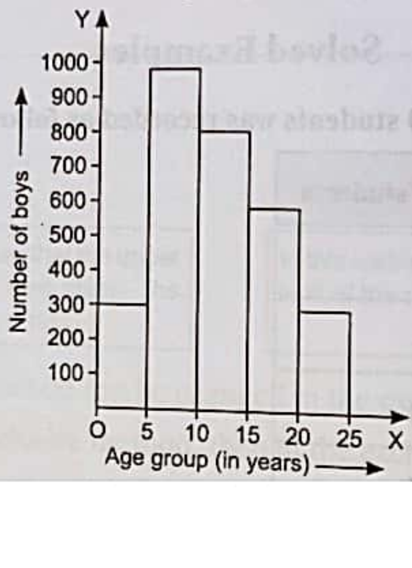
Heights (in cm)	Heights (in cm) after adjustment factor	Frequency	Cumulative frequency
121 – 130	120.5 – 130.5	12	12
131 – 140	130.5 – 140.5	16	28
141 – 150	140.5 – 150.5	30	58
151 – 160	150.5 – 160.5	20	78
161 – 170	160.5 – 170.5	14	92
171 – 180	170.5 – 180.5	8	100



Q 3. **Represent the following distribution by means of a histogram:**

Age group (in years)	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Number of boys	300	980	800	580	290

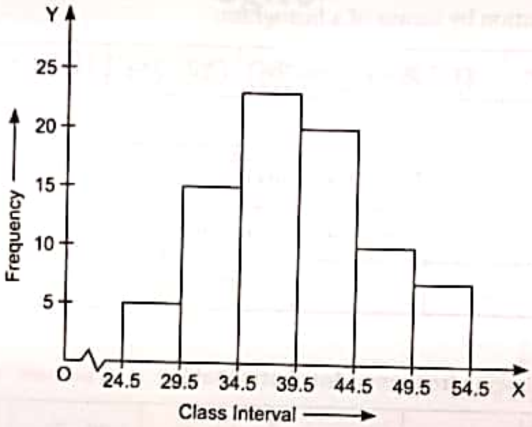
Sol. Represent the class interval on the x-axis and frequency on the y-axis as shown in figure:



Q 4. **Represent the following distribution by means of a histogram:**

Class Interval	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
Frequency	5	15	23	20	10	7

Sol. Change the class interval in exclusive method and then draw histogram. So, the new class interval in exclusive method will be as 24.5–29.5, 29.5–34.5, 34.5–39.5, 39.5–44.5, 44.5–49.5, 49.5–54.5



Q 5. Represent the following frequency distribution by means of a histogram:

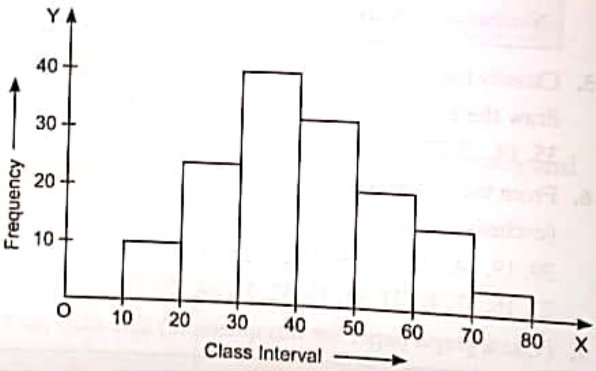
x	15	25	35	45	55	65	75
f	10	24	40	32	20	14	4

Sol. Change the mid-values (x) in the form of class intervals and then draw histogram.

Difference between IInd and Ist mid-value $= 25 - 15 = 10$

$\therefore$ Class-size, $h = 10 \Rightarrow \frac{h}{2} = 5$

Class Interval	Frequency
10 – 20	10
20 – 30	24
30 – 40	40
40 – 50	32
50 – 60	20
60 – 70	14
70 – 80	4



Mean, Median and Mode

[CHAPTER – 24]

1. Find the mean of all prime numbers between 51 and 80.

Sol. Prime numbers between 51 and 80 are 53, 59, 61, 67, 71, 73, 79

$$\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{N} = \frac{53+59+61+67+71+73+79}{7} = \frac{463}{7} = 66.14$$

2. The mean of 40 observations is 160. It was detected on rechecking that the value 165 was wrongly copied as 125. Find the correct mean.

Sol.
$$\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{n}$$

$\Rightarrow$
$$160 = \frac{\Sigma x_i}{40} \Rightarrow \Sigma x_i = 160 \times 40 = 6400$$

$$\text{Correct } \Sigma x_i = \text{incorrect } \Sigma x_i - \text{incorrect value} + \text{correct value}$$
$$= 6400 - 125 + 165 = 6440$$

$$\text{Correct mean} = \frac{\text{Correct } \Sigma x_i}{n} = \frac{6440}{40} = 161.$$

3. Calculate the mean of the following frequency distribution by direct method:

x	4	6	9	10	15
f	5	10	10	7	8

Sol.

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
Total	$\Sigma f_i = 40$	$\Sigma f_i x_i = 360$

From the table, $\Sigma f_i = 40$ and $\Sigma f_i x_i = 360$

So,
$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{360}{40} = 9$$

4. Calculate the mean of the following data (Using short-cut Method).

C.I.	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	8	10	2	4

Sol.

C.I.	Mid-value (x_i)	Frequency (f_i)	$d_i = x_i - A$	$f_i d_i$
0 – 10	5	6	−20	−120] = −200
10 – 20	15	8	−10	−80
20 – 30	$\boxed{A = 25}$	10	0	0
30 – 40	35	2	10	20] = 100
40 – 50	45	4	20	80
Total		$\Sigma f_i = 30$		$\Sigma f_i d_i = -100$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 25 + \frac{(-100)}{30} = 25 - 3.33 = 21.67$$

5. Calculate the mean of the following frequency distribution by step-deviation method:

x	15	20	25	30	35	40	45	50	55
f	5	8	11	20	23	18	13	3	1

Sol.

x_i	f_i	$u_i = \frac{x_i - A}{h} (h = 5)$	$f_i u_i$
15	5	−4	−20
20	8	−3	−24
25	11	−2	−22
30	20	−1	−20
$\boxed{A = 35}$	23	0	0
40	18	1	18
45	13	2	26
50	3	3	9
55	1	4	4
Total	$\Sigma f_i = 102$		$\Sigma f_i u_i = -29$

Here, $A = 35, \Sigma f_i = 102, h = 5, \Sigma f_i u_i = -29$

So, $$\text{Mean } (\bar{x}) = A + h\frac{\Sigma f_i u_i}{\Sigma f_i}$$

$$= 35 + 5 \times \frac{(-29)}{102} = 35 - \frac{145}{102}$$

$$= 35 - 1.42 = 33.58$$

6. If the mean of the following frequency distribution is 7.5, find the value of p.

x	3	5	7	9	11	13
f	6	8	15	p	8	4

Sol.

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	p	$9p$
11	8	88
13	4	52
Total	$\Sigma f_i = 41 + p$	$\Sigma f_i x_i = 303 + 9p$

Given, mean $(\bar{x}) = 7.5$

Here, $\Sigma f_i = 41 + p$ and $\Sigma f_i x_i = 303 + 9p$

So, $$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 7.5 = \frac{303 + 9p}{41 + p}$$

$$\Rightarrow \quad \frac{75}{10} = \frac{303 + 9p}{41 + p}$$

$$\Rightarrow \quad 3030 + 90p = 3075 + 75p$$

$$\Rightarrow \quad 90p - 75p = 3075 - 3030$$

$$\Rightarrow \quad 15p = 45 \Rightarrow p = 3$$

7. **The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in the classes 20 - 40 and 60–80 are missing. Find the value of f_1 and f_2.**

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	32	f_2	19	120

Sol.

Class interval	Mid value (x_i)	f_i	$f_i x_i$
0 – 20	$\frac{0+20}{2} = 10$	17	170
20 – 40	30	f_1	$30f_1$
40 – 60	50	32	1600
60 – 80	70	f_2	$70f_2$
80 – 100	90	19	1710
Total		$\Sigma f_i = 120$	$\Sigma f_i x_i = 3480 + 30f_1 + 70f_2$

So, $\quad$ Mean ($\bar{x}$) $= \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 50 = \frac{3480 + 30f_1 + 70f_2}{120}$

$\Rightarrow \quad 6000 = 3480 + 30f_1 + 70f_2$

Also, $\quad 120 = 68 + f_1 + f_2 \Rightarrow f_1 + f_2 = 52$

From (*i*),we get

$$30f_1 + 70f_2 = 2520$$

$\Rightarrow \quad 3f_1 + 7f_2 = 252$

From (*ii*), we get

$$f_1 + f_2 + 68 = 120$$

$\Rightarrow \quad f_1 + f_2 = 52$

On multiplying (*iii*) by 1 and (*ii*) by 3, we get

$$3f_1 + 7f_2 = 252$$
$$3f_1 + 3f_2 = 156$$
$$- \quad - \quad\quad -$$

$$4f_2 = 96$$

$\Rightarrow \quad f_2 = 24$

Putting $f_2 = 24$ in (*ii*), we get

$f_1 + f_2 = 52 \Rightarrow f_1 + 24 = 52 \Rightarrow f_1 = 28$

8. **Find the mean of the following cumulative frequency table:**

Marks	More than 0	More than 10	More than 20	More than 30	More than 40	More than 50
Number of students	60	56	40	20	10	3

Sol.

Marks	Number of students (c.f.)	Mid-value (x_i)	f_i	f_ix_i
More than 0	60	$\frac{0+10}{2} = 5$	4	20
More than 10	56	15	16	240
More than 20	40	25	20	500
More than 30	20	35	10	350
More than 40	10	45	7	315
More than 50	3	55	3	165
Total			$\Sigma f_i = 60$	$\Sigma f_ix_i = 1590$

we have $\Sigma f_i = 60$ and $\Sigma f_ix_i = 1590$

So, $\text{Mean } (\bar{x}) = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{1590}{60} = 26.5$

9. The mean of 10 numbers is 20. If 5 is subtracted from every number, what will be the new mean?

Sol. $\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{n} \Rightarrow 20 = \frac{\Sigma x_i}{10} \Rightarrow \Sigma x_i = 200$

Each number is subtracted by 5, so 10 numbers are subtracted by $5 \times 10 = 50$

So, $\text{new } \Sigma x_i = 200 - 50 = 150$

$$\text{New mean } (\bar{x}_i) = \frac{\text{new } \Sigma x_i}{n} = \frac{150}{10} = 15$$

10. The mean of 8 numbers is 15. If each number is multiplied by 2, what will be the new mean?

Sol. $\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{n} \Rightarrow 15 = \frac{\Sigma x_i}{8} \Rightarrow \Sigma x_i = 120$

When each number is multiplied by 2, then

$$\text{new } \Sigma x_i = 120 \times 2 = 240$$

$$\text{New mean} = \frac{\text{new } \Sigma x_i}{n} = \frac{240}{8} = 30$$

11. The mean weight of a class of 35 students is 45 kg. If the weight of the teacher is included, the mean weight increases by 500 g. Find the weight of the teacher.

Sol. $\text{Mean } (\bar{x}) = \frac{\Sigma x_i}{n}$

$\Rightarrow \quad 45 = \frac{\Sigma x_i}{35} \Rightarrow \Sigma x_i = 45 \times 35 = 1575 \text{ kg}$

Now, $n = 35 + 1 = 36$

New mean $= 45 \text{ kg} + 500 \text{ g} = 45.5 \text{ kg}$

Now, new $\Sigma x_i = 45.5 \times 36 = 1638$

$\therefore$ Weight of teacher $= 1638 \text{ kg} - 1575 \text{ kg} = 63 \text{ kg}$

12. Marks obtained by 40 students in a short assessment is given below, where *a* and *b* are two missing data.

Marks	5	6	7	8	9
Number of students	6	*a*	16	13	*b*

If the mean of the distribution is 7.2, find *a* and *b*.

Sol.

Marks (x_i)	Number of students (f_i)	$f_i x_i$
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
Total	$\Sigma f_i = 35 + a + b$	$\Sigma f_i x_i = 246 + 6a + 9b$

Now, $35 + a + b = 40 \Rightarrow a + b = 5$ ... (*i*)

Also, Mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 7.2 = \dfrac{246 + 6a + 9b}{40}$

$\Rightarrow \dfrac{72}{10} \times 40 = 246 + 6a + 9b \Rightarrow 288 = 3(82 + 2a + 3b)$

$\Rightarrow 96 = 82 + 2a + 3b \Rightarrow 2a + 3b = 14$ ... (*ii*)

On multiplying equation (*i*) by 2, we get

$\Rightarrow 2a + 2b = 10$ ... (*iii*)

On subtracting equation (*iii*) from (*ii*), we get

$b = 4$

From (*i*), we get

$a + 4 = 5 \Rightarrow a = 1 \Rightarrow a = 1$ and $b = 4$

13. The mean of the following distribution is 52 and the frequency of class interval 30 – 40 is 'f'. Find 'f'.

Class interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	5	3	f	7	2	6	13

Sol.

Class interval	Mid-value (x_i)	Frequency (f_i)	$f_i x_i$
10 – 20	15	5	75
20 – 30	25	3	75
30 – 40	35	f	$35f$
40 – 50	45	7	315
50 – 60	55	2	110
60 – 70	65	6	390
70 – 80	75	13	975
Total		$\Sigma f_i = 36 + f$	$\Sigma f_i x_i = 1940 + 35f$

$$\text{Mean} = \frac{\Sigma f x_i}{\Sigma f_i} \Rightarrow 52 = \frac{1940 + 35f}{36 + f}$$

$\Rightarrow 1872 + 52f = 1940 + 35f$

$\Rightarrow 52f - 35f = 1940 - 1872$

$\Rightarrow 17f = 68 \Rightarrow f = 4$

14. Using step-deviation method, calculate the mean marks of the following distribution.

Sol.

Class interval	Mid-value (x_i)	f_i	$d_i = x_i - A$	$u_i = \frac{d_i}{h}$ ($h = 5$)	$f_i \times u_i$
50 – 55	52.5	5	–15	–3	–15
55 – 60	57.5	20	–10	–2	–40
60 – 65	62.5	10	–5	–1	–10
65 – 70	$\boxed{A = 67.5}$	10	0	0	0
70 – 75	72.5	9	5	1	9
75 – 80	77.5	6	10	2	12
80 – 85	82.5	12	15	3	36
85 – 90	87.5	8	20	4	32
Total		$\Sigma f_i = 80$			$\Sigma f_i u_i = 89 - 65 = 24$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 67.5 + \frac{24}{80} \times 5 = 67.5 + \frac{24}{16} = 67.5 + 1.5 = 69.0$$

15. **The monthly income of a group of 320 employees in a company is given below:**

Monthly income (in ₹)	Number of employees
6000 – 7000	20
7000 – 8000	45
8000 – 9000	65
9000 – 10000	95
10000 – 11000	60
11000 – 12000	30
12000 – 13000	5

Draw an ogive for the given distribution on a graph sheet taking 1 cm = ₹ 1,000 on one axis and 1 cm = 50 employees on the other axis. From the graph, determine:

(i) the median wage.

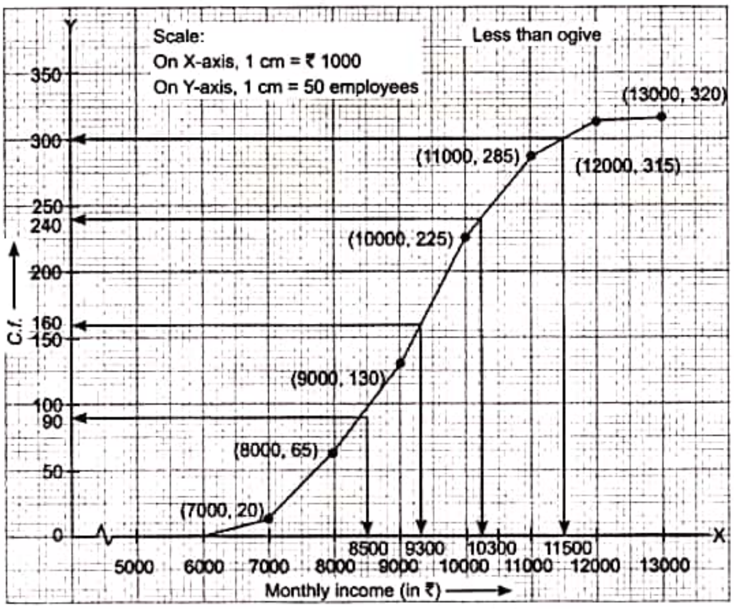
(ii) the number of employees whose income is below ₹ 8,500.

(iii) if the salary of a senior empolyee is above ₹ 11, 500, find the number of senior employees in the company.

(iv) the upper quartile.

Sol.

Monthly income (in ₹)	Number of employees (f)	C.f.
6000 – 7000	20	20
7000 – 8000	45	65
8000 – 9000	65	130
9000 – 10000	95	225
10000 – 11000	60	285
11000 – 12000	30	315
12000 – 13000	5	320
	$n = 320$	



(i) Median wage $= \dfrac{n}{2}^{\text{th}}$ term $= \dfrac{320}{2}^{\text{th}}$ term $= 160^{\text{th}}$ term $=$ ₹ 9300

(ii) Number of employees whose income is below ₹ 8500 = 90

(iii) Number of senior employees in the company = 320 – 300 = 20

(iv) Upper quartile $= \dfrac{3n}{4}^{\text{th}}$ term $= \dfrac{3 \times 320}{4}^{\text{th}}$ term $= 240^{\text{th}}$ term $=$ ₹ 10,300

16. **Marks obtained by 200 students in an examination are given below:**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Number of students	5	11	10	20	28	37	40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine:

(*i*) The median marks.

(*ii*) The number of students who failed, if minimum marks required to pass is 40.

(*iii*) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination.

Sol.

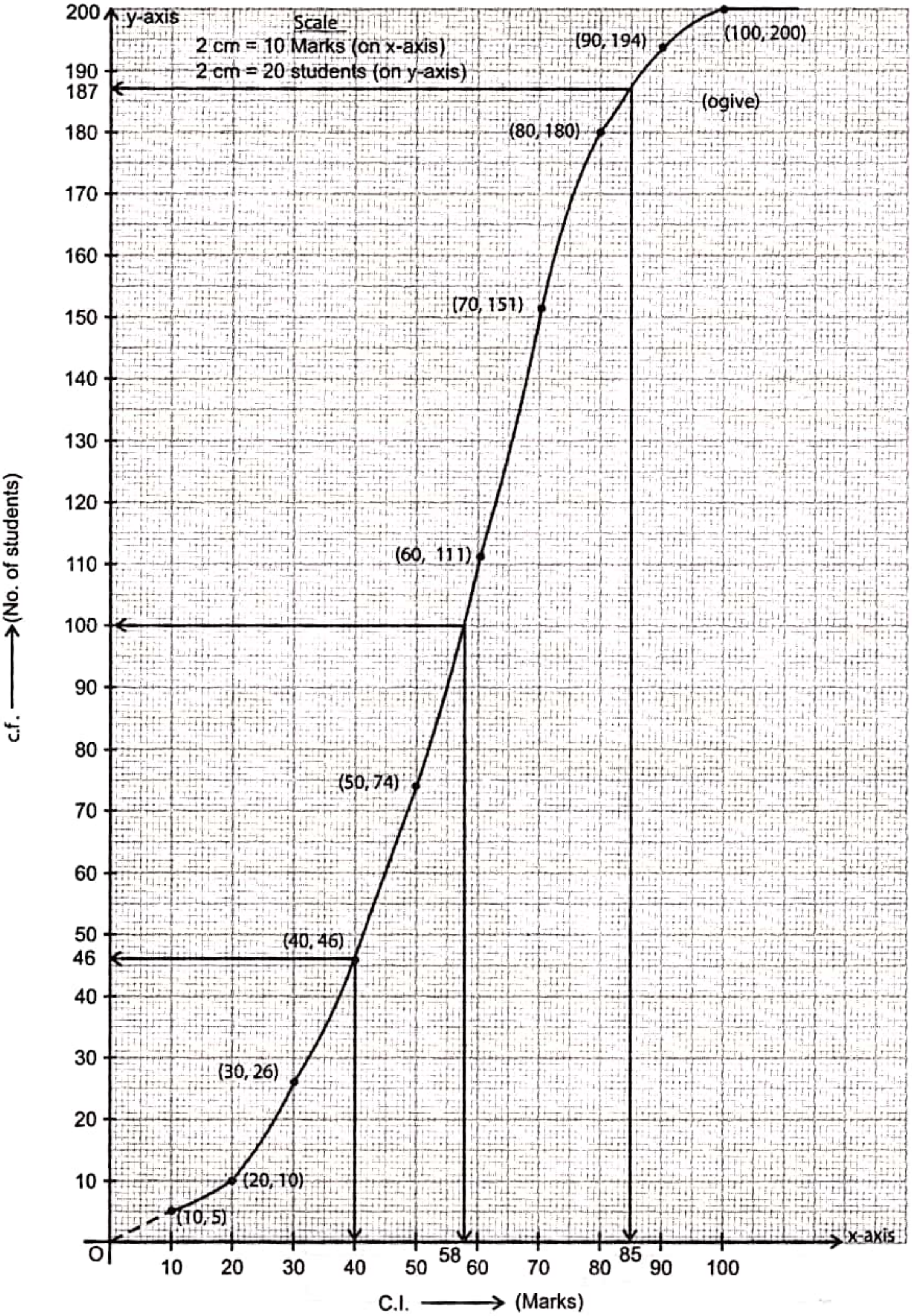
Marks	Number of students (f)	C.f.
0 – 10	5	5
10 – 20	11	16
20 – 30	10	26
30 – 40	20	46
40 – 50	28	74
50 – 60	37	111
60 – 70	40	151
70 – 80	29	180
80 – 90	14	194
90 – 100	6	200
	$n = 200$	

(*i*) Median $= \frac{N}{2} = \frac{200}{2} = 100$

So, Median = 58 Marks

(*ii*) 46 students

(*iii*) $200 - 187 = 13$ students



y-axis
Scale
2 cm = 10 Marks (on x-axis)
2 cm = 20 students (on y-axis)
(ogive)
(100, 200)
(90, 194)
(80, 180)
(70, 151)
(60, 111)
(50, 74)
(40, 46)
(30, 26)
(20, 10)
(10, 5)
200
190
187
180
170
160
150
140
130
120
110
100
90
80
70
60
50
46
40
30
20
10
O
10
20
30
40
50
58
60
70
80
85
90
100
x-axis
c.f. ⟶ (No. of students)
C.I. ⟶ (Marks)

17. **The table below shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.**

Scores obtained	Number of shooters
0 – 10	5
10 – 20	9
20 – 30	16
30 – 40	22
40 – 50	26
50 – 60	18
60 – 70	11
70 – 80	6
80 – 90	4
90 – 100	3

Use your ogive to estimate:

(i) The Median. **(ii) The inter quartile range.**

(iii) The number of shooters who obtained more than 75% scores.

Sol.

Scores obtained	Number of shooters	C.f.
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113
80 – 90	4	117
90 – 100	3	120
	$n = 120$	

(i) $n = 120$ (even)

The position of median is given by $\frac{n}{2}^{\text{th}}$ term $= \frac{120}{2}^{\text{th}}$ term $= 60$th term

So, from the graph, median = 43 scores

(ii) The position of Q_1 is given by $\frac{n}{4}^{\text{th}}$ term $= \frac{120}{4}^{\text{th}}$ term $= 30$th term

So, from the graph, $Q_1 = 30$ scores

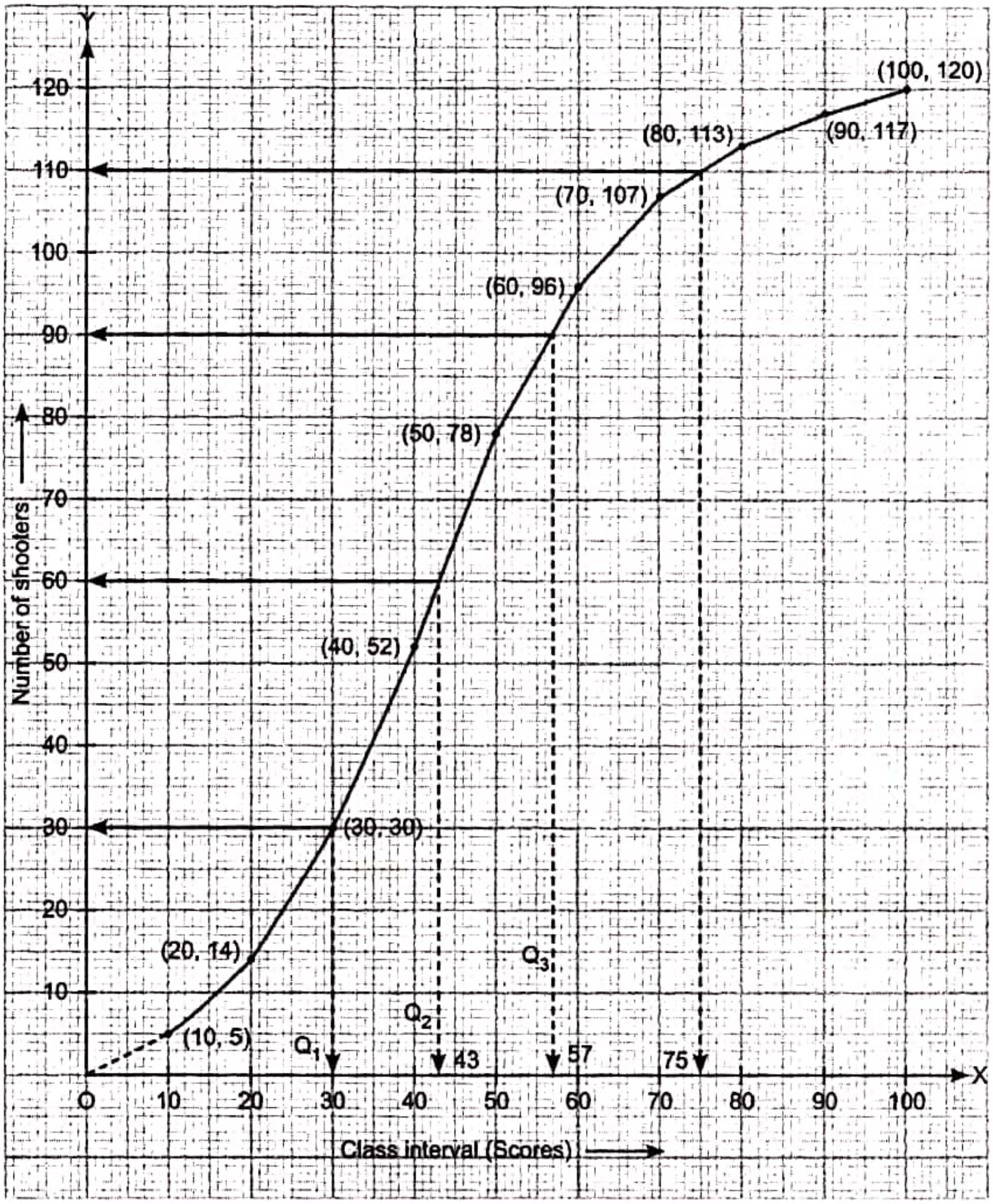
The position of Q_3 is given by $\frac{3n}{4} = \frac{3 \times 120}{4} = 90$

So, from the graph, $Q_3 = 57$ scores

Now, interquartile range $= Q_3 - Q_1$

$= 57 - 30 = 27$ scores

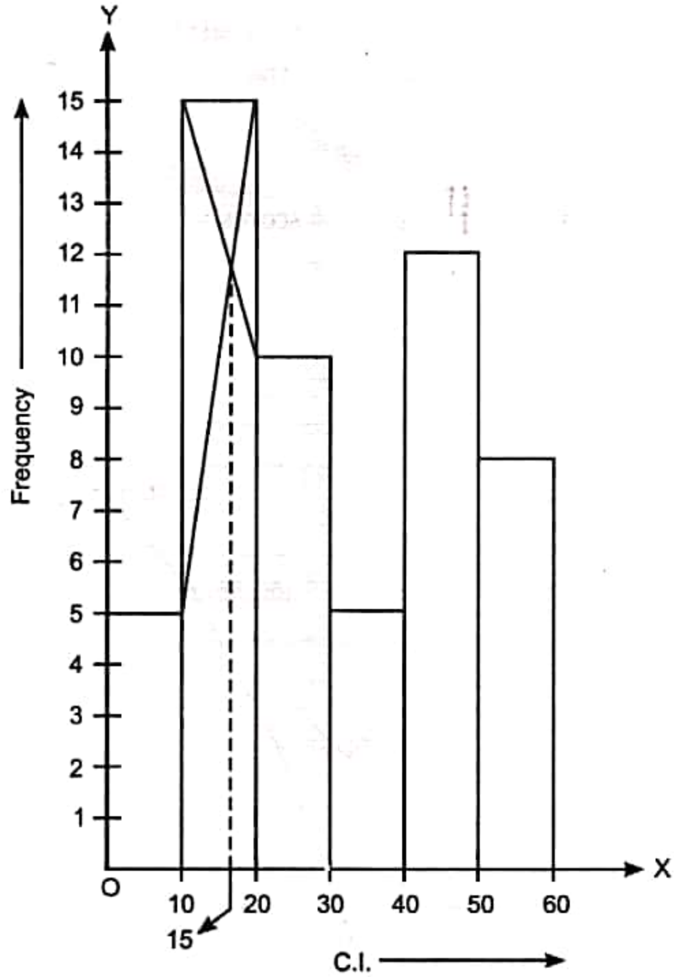
(iii) Number of shooters who obtained more than 75% scores $= 120 - 110 = 10$



18. Draw histogram of the following fequency distribution and using it, calculate the mode.

C.I.	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	15	10	5	12	8

Sol.



So, Mode = 15

Probability
[CHAPTER – 25]

Ex.1. **(i) Which one of the following cannot be the probability of an event?**

(a) $\frac{2}{3}$ (b) 15% (c) $\frac{3}{2}$ (d) 0.0003

(ii) For an event P(E) = 0.85, find P(not E).

Sol. (i) $\because \quad 0 \le P(E) \le 1$

$\therefore$ (c) part cannot be the probability of an event.

(ii) P(not E) = 1 − P(E) = 1 − 0.85 = 0.15

Ex.2. Chikoo and Gulu are friends. Find the probability that both of them have (i) same birthdays (ii) different birthdays. (Ignore a leap year).

Sol. (i) Both can have same birthday on any one day of the year.

Thus $P(\text{same birthday}) = \frac{1}{365}$

(ii) P(different birthdays) = 1 − P(same birthday)

$= 1 - \frac{1}{365} = \frac{364}{365}$

Ex.3. A coin is tossed once. Find the probability of getting,

(i) a head (ii) a tail

(iii) not a head (iv) not a tail

Sol. Sample space is {H, T}

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

(i) $P(\text{a head}) = \frac{1}{2}$, (ii) $P(\text{a tail}) = \frac{1}{2}$

(iii) $P(\text{not a head}) = P(\text{a tail}) = \frac{1}{2}$

(iv) $P(\text{not a tail}) = P(\text{a head}) = \frac{1}{2}$

Ex.4. A bag contains 5 red, 4 blue and 3 green balls. One ball is drawn at random. Find the probability that the ball drawn is:

(i) blue (ii) not red

(iii) either red or green (iv) black

(v) red and green.

Sol. Total number of balls = 5 + 4 + 3 = 12

(i) Number of favourable outcomes = 4

Total number of possible outcomes = 12

$\therefore \quad P(\text{blue}) = \frac{4}{12} = \frac{1}{3}$

(ii) $P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{5}{12} = \frac{7}{12}$

(iii) For either red or green balls, we have favourable outcomes = 5 + 3 = 8

$\therefore \quad P(\text{either red or green}) = \frac{8}{12} = \frac{2}{3}$

(iv) There is no black ball in the bag, thus P(black) = 0 (impossible event)

(v) P (red and green) = 0 (impossible event). (As every ball has one colour only).

Ex.5. Two coins are tossed once. Find the probability of getting:

(i) 2 heads

(ii) at least 1 tail.

Sol. Sample space is {HH, HT, TH, TT}

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

(i) $P(\text{2 heads}) = \frac{1}{4}$

(ii) $P(\text{at least 1 tail}) = \frac{3}{4}$

Ex.6. **From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random. What is the probability that the card drawn is**

(i) a face card (King, Jack or Queen)?

(ii) an even numbered red card?

Sol. Number of cards having a number which is multiple of 3 = 12

Number of cards left = 52 – 12 = 40

(i) P (a face card) = $\frac{12}{40} = \frac{3}{10} = 0.3$

(ii) P(an even numbered red card) = $\frac{10}{40} = \frac{1}{4}$

Ex.7. **A bag contains 8 green balls and some red balls. If the probability of drawing a red ball is half that of a green ball, then find the number of red balls in the bag.**

Sol. Number of green balls = 8

Let number of red balls be x

Total number of balls be $(8 + x)$

$$\text{P(red ball)} = \frac{x}{8+x},$$

$$\text{P(green ball)} = \frac{8}{8+x}$$

According to the question,

$$\text{P(red ball)} = \frac{1}{2}\text{P(green ball)}$$

$$\Rightarrow \quad \frac{x}{8+x} = \frac{1}{2} \times \frac{8}{(x+8)}$$

$$\Rightarrow \quad x = 4$$

Ex.8. **(i) A bag contains 20 balls out of which x balls are blue. If one ball is drawn at random, find the probability of drawing a blue ball.**

(ii) If 10 more blue balls are put in the bag mentioned in part (i) and a ball is drawn at a random then the probability of getting a blue ball is double the probability obtained in part (i). Find x.

Sol. (i) P(a blue ball) = $\frac{x}{20}$

(ii) Now, total number of balls = 20 + 10 = 30

Number of blue balls = $x + 10$

$$\text{P(blue ball)} = \frac{x+10}{30}$$

According to the question

$$\frac{x+10}{30} = 2 \times \frac{x}{20}$$

$\Rightarrow$ $x = 5$

Ex.9. A die is thrown once. Find the probability of getting:

(i) a number divisible by 3

(ii) a factor of 6

(iii) either 3 or 4

(iv) neither 2 nor 4

(v) a number from 1 to 6

(vi) a number 7

Sol. Sample space: {1,2,3,4,5,6}

(i) P (a number divisible by 3) = $\frac{2}{6} = \frac{1}{3}$

(ii) P (a factor of 6) = $\frac{4}{6} = \frac{2}{3}$ (1,2,3 and 6 are factors of 6)

(iii) P(either 3 or 4) = $\frac{2}{6} = \frac{1}{3}$

(iv) P(neither 2 nor 4) = $\frac{4}{6} = \frac{2}{3}$

(v) P(a number from 1 to 6) = 1 (sure event)

(vi) P (a number 7) = 0 (impossible event)

Ex.10. An alphabet is chosen from english alphabet. Find the probability that the alphabet chosen is

(i) a vowel **(ii) a consonant** **(iii) h**

Sol. a, e, i, o and u are 5 vowels and 21 of the alphabet are consonants.

(i) P (a vowel) = $\frac{5}{26}$

(ii) P (a consonant) = $\frac{21}{26}$

(iii) P (h) = $\frac{1}{26}$

Ex.11. From a pack of 52 playing cards jack, queen and king of clubs are removed and then well shuffled. From the remaining cards a card is drawn. Find the probability of getting :

(i) a face card

(ii) a club

(iii) a black card

(iv) a non face card

(v) a king

(vi) a red face card

Sol. Number of cards removed = 3 (face cards of clubs)

Remaining number of cards = 52 − 3 = 49

<u>First Mock test over</u>

Model QUESTION BANK

Second Mock Test

CLASS – X

MATHEMATICS

Ratio and Proportion

[CHAPTER – 7]

1. If $(2x + 3) : (5x - 38)$ be duplicate ratio of $\sqrt{5} : \sqrt{6}$, find the value of x.

Sol. Put $\frac{2x+3}{5x-38} = \frac{5}{6}$

By cross-multiplication,

$\Rightarrow$ $12x + 18 = 25x - 190 \Rightarrow 12x - 25x = -190 - 18$

$\Rightarrow$ $-13x = -208 \Rightarrow x = 16$

2. If $12(2x^2 - 3y^2) = 8x^2 - 11y^2$, find $(4x + y) : (4x - y)$.

Sol. $12(2x^2 - 3y^2) = 8x^2 - 11y^2$

$\Rightarrow$ $16x^2 = 25y^2 \Rightarrow 4x = 5y \Rightarrow x = \frac{5y}{4}$

Put $x = \frac{5y}{4}$ in $(4x + y) : (4x - y)$, we get

$\left(4 \times \frac{5y}{4} + y\right) : \left(4 \times \frac{5y}{4} - y\right) = 6y : 4y$

$\Rightarrow$ $6 : 4 \Rightarrow 3 : 2$

3. If $\frac{3x - 4y}{2x - 3y} = \frac{5x - 6y}{4x - 5y}$, find $x : y$.

Sol. $\frac{3x - 4y}{2x - 3y} = \frac{5x - 6y}{4x - 5y}$

By cross multiplication, we get

$12x^2 - 15xy - 16xy + 20y^2 = 10x^2 - 15xy - 12xy + 18y^2$

$\Rightarrow$ $12x^2 - 31xy + 20y^2 = 10x^2 - 27xy + 18y^2$

$\Rightarrow$ $10x^2 - 27xy + 18y^2 - 12x^2 + 31xy - 20y^2 = 0$

$\Rightarrow$ $-2x^2 + 4xy - 2y^2 = 0$

$\Rightarrow$ $x^2 - 2xy + y^2 = 0$

$\Rightarrow$ $(x - y)^2 = 0$

$\Rightarrow$ $x - y = 0$

$\Rightarrow$ $x = y$

Hence, $x : y = 1 : 1$.

4. What must be added to each term of 2 : 5, so that it may be equal to 5 : 6?

Sol. Let x be added to each term of 2 : 5.

So, $\frac{2+x}{5+x} = \frac{5}{6}$

By cross multiplication, we get

$12 + 6x = 25 + 5x \Rightarrow 6x - 5x = 25 - 12$

$\Rightarrow$ $x = 13$.

5. If $(6a^2 - ab) : (2ab - b^2) = 6 : 1$, find $a : b$.

Sol. $\frac{6a^2 - ab}{2ab - b^2} = \frac{6}{1}$

By cross-multiplication, we get

$\Rightarrow$ $6a^2 - ab = 12ab - 6b^2$

$\Rightarrow$ $6a^2 - 13ab + 6b^2 = 0 \Rightarrow (2a - 3b)(3a - 2b) = 0$

So, either $2a - 3b = 0$ or $3a - 2b = 0$

Hence, either $\frac{a}{b} = \frac{3}{2}$ or $\frac{a}{b} = \frac{2}{3}$

6. Are the numbers 6, 10, 14 and 22 in proportion? If no, then what must be added to each of them, to make them in proportion.

Sol. Let x be added to each term, to make them in proportion.

So, $$\frac{6+x}{10+x} = \frac{14+x}{22+x}$$

$\Rightarrow$ $$132 + 6x + 22x + x^2 = 140 + 10x + 14x + x^2$$

$\Rightarrow$ $$132 + 28x = 140 + 24x$$

$\Rightarrow$ $$4x = 8 \Rightarrow x = 2$$

7. There are 36 members on a student council in a school and the ratio of the number of boys to the number of girls is 3 : 1. How many more girls should be added to the council so that the ratio of number of boys to the number of girls will be 9 : 5?

Sol. Let the number of boys be $3x$

and the number of girls be x

So, $$3x + x = 36 \Rightarrow x = 9$$

Hence, number of boys = 27 and number of girls = 9

Let 'a' more girls be added, then $\frac{27}{9+a} = \frac{9}{5}$

$\Rightarrow$ $$81 + 9a = 135 \Rightarrow 9a = 135 - 81$$

$\Rightarrow$ $$9a = 54 \Rightarrow a = 6$$

So, 6 more girls should be added.

8. In a regiment, the ratio of number of officers to the number of soldiers was 3 : 31 before a battle. In the battle 6 officers and 22 soldiers were killed. The ratio between the number of officers and the number of soldiers now is 1 : 13. Find the number of officers and soldiers in the regiment before the battle.

Sol. Before battle,

Let the number of officers be $3x$ and the number of soldiers be $31x$

Now after battle $$\frac{3x-6}{31x-22} = \frac{1}{13}$$

$\Rightarrow$ $$39x - 78 = 31x - 22$$

$\Rightarrow$ $$39x - 31x = 78 - 22$$

$\Rightarrow$ $$8x = 56 \Rightarrow x = 7$$

So, number of officers= $3x = 3 \times 7 = 21$

number of soldiers= $31x = 31 \times 7 = 217$

9. **If $\frac{a}{b}=\frac{c}{d}$, prove that: $(2a+3b)(2c-3d)=(2a-3b)(2c+3d)$.**

Sol. Put $\frac{a}{b}=\frac{c}{d}=k$, so $a=bk$, $c=dk$ put the values of a and c in both the sides,

$$\begin{aligned}\text{LHS} &= (2a+3b)(2c-3d) = (2bk+3b)(2dk-3d)\\ &= b(2k+3)\,d(2k-3)\\ &= bd\,(4k^2-9)\\ \text{RHS} &= (2a-3b)(2c+3d) = (2bk-3b)(2dk+3d)\\ &= b\,(2k-3)\,d(2k+3)\\ &= bd\,(4k^2-9)\end{aligned}$$

So, LHS = RHS Hence proved.

10. **If $\frac{11a^2+13b^2}{11c^2+13d^2}=\frac{11a^2-13b^2}{11c^2-13d^2}$, prove that a, b, c and d are in proportion.**

Sol.
$$\frac{11a^2+13b^2}{11c^2+13d^2} = \frac{11a^2-13b^2}{11c^2-13d^2}$$

By alternendo, we get

$$\frac{11a^2+13b^2}{11a^2-13b^2} = \frac{11c^2+13d^2}{11c^2-13d^2}$$

By componendo and dividendo, we get

$$\frac{11a^2+13b^2+11a^2-13b^2}{11a^2+13b^2-11a^2+13b^2} = \frac{11c^2+13d^2+11c^2-13d^2}{11c^2+13d^2-11c^2+13d^2}$$

$$\Rightarrow \quad \frac{22a^2}{26b^2} = \frac{22c^2}{26d^2} \Rightarrow \frac{a^2}{b^2}=\frac{c^2}{d^2}$$

By taking square root of both sides, we get

$$\Rightarrow \quad \frac{a}{b}=\frac{c}{d}$$

So, a, b, c and d are in proportion. Hence proved.

11. **If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$, prove that each ratio is equal to $\frac{x+y+z}{a+b+c}$.**

Sol. Each ratio $= \dfrac{x+y+z}{b+c-a+c+a-b+a+b-c}$

$$= \frac{x+y+z}{a+b+c}, \qquad \left(\text{Each ratio} = \frac{\text{Sum of antecedents}}{\text{Sum of consequents}}\right)$$ Hence proved.

12. **If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, show that each ratio is equal to $\frac{1}{2}$ or -1.**

Sol. Each ratio $= \dfrac{a+b+c}{b+c+c+a+a+b} = \dfrac{a+b+c}{2(a+b+c)} = \dfrac{1}{2}$

If $a+b+c=0$, then

$$\frac{a}{b+c} = \frac{a}{a+b+c-a} = \frac{a}{0-a} = \frac{a}{-a} = -1$$

Similarly, all other terms will be equal to -1.

13. **If $\frac{b+c-a}{y+z-x}=\frac{c+a-b}{z+x-y}=\frac{a+b-c}{x+y-z}$, then prove that each ratio is equal to $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}$.**

Sol. Each ratio $= \dfrac{(b+c-a)+(c+a-b)+(a+b-c)}{(y+z-x)+(z+x-y)+(x+y-z)} = \dfrac{a+b+c}{x+y+z}$

Now, $\dfrac{b+c-a}{y+z-x} = \dfrac{a+b+c}{x+y+z}$

$\Rightarrow$ $\dfrac{b+c-a}{a+b+c} = \dfrac{y+z-x}{x+y+z}$ [By alternendo]

$\Rightarrow$ $\dfrac{(b+c-a)-(a+b+c)}{a+b+c} = \dfrac{(y+z-x)-(x+y+z)}{x+y+z}$ [By dividendo]

$\Rightarrow$ $\dfrac{a}{a+b+c} = \dfrac{x}{x+y+z}$

$\Rightarrow$ $\dfrac{a}{x} = \dfrac{a+b+c}{x+y+z}$ ...(*i*)

Now, $\dfrac{c+a-b}{z+x-y} = \dfrac{a+b+c}{x+y+z}$

By alternendo

$$\frac{c+a-b}{a+b+c} = \frac{z+x-y}{x+y+z}$$

$\Rightarrow$ $\dfrac{-2b}{a+b+c} = \dfrac{-2y}{x+y+z}$

So, $\dfrac{b}{y} = \dfrac{a+b+c}{x+y+z}$ ...(*ii*)

Now, $\dfrac{a+b-c}{x+y-z} = \dfrac{a+b+c}{x+y+z}$

By alternendo

$$\frac{a+b-c}{a+b+c} = \frac{x+y-z}{x+y+z}$$

By dividendo

$$\frac{a+b-c-a-b-c}{a+b+c} = \frac{x+y-z-x-y-z}{x+y+z}$$

$\Rightarrow$ $\dfrac{-2c}{a+b+c} = \dfrac{-2z}{x+y+z}$

$\Rightarrow$ $\dfrac{c}{z} = \dfrac{a+b+c}{x+y+z}$ ...(*iii*)

From eq. (*i*) and (*ii*) and (*iii*), we get

$\dfrac{a}{x} = \dfrac{b}{y} = \dfrac{c}{z}$.

14. **If $(a - 2b - 3c + 4d)(a + 2b + 3c + 4d) = (a + 2b - 3c - 4d)(a - 2b + 3c - 4d)$, show that $2ad = 3bc$.**

Sol. Write as $$\frac{(a-2b-3c+4d)}{(a+2b-3c-4d)} = \frac{(a-2b+3c-4d)}{(a+2b+3c+4d)}$$

Now, using componendo and dividendo, we get

$$\frac{a-2b-3c+4d+a+2b-3c-4d}{a-2b-3c+4d-a-2b+3c+4d} = \frac{a-2b+3c-4d+a+2b+3c+4d}{a-2b+3c-4d-a-2b-3c-4d}$$

$\Rightarrow$ $$\frac{2a-6c}{-4b+8d} = \frac{2a+6c}{-4b-8d}$$

$\Rightarrow$ $$\frac{2(a-3c)}{-4(b-2d)} = \frac{2(a+3c)}{-4(b+2d)}$$

$\Rightarrow$ $$\frac{a-3c}{b-2d} = \frac{a+3c}{b+2d}$$

By alternendo

$$\frac{a-3c}{a+3c} = \frac{b-2d}{b+2d}$$

By componendo and dividendo

$$\frac{a-3c+a+3c}{a-3c-a-3c} = \frac{b-2d+b+2d}{b-2d-b-2d}$$

$\Rightarrow$ $$\frac{2a}{-6c} = \frac{2b}{-4d} \Rightarrow \frac{a}{3c} = \frac{b}{2d}$$

$\Rightarrow$ $$2ab = 3bc$$

15. **Using componendo and dividendo, find the value of x : $\dfrac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9.$**

Sol. Given, $\dfrac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = 9$

By using componendo and dividendo, we get,

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}+\sqrt{3x+4}-\sqrt{3x-5}}{\sqrt{3x+4}+\sqrt{3x-5}-\sqrt{3x+4}+\sqrt{3x-5}} = \frac{9+1}{9-1}$$

$\Rightarrow$ $$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$\Rightarrow$ $$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides, we get

$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

$\Rightarrow$ $$75x - 125 = 48x + 64$$

$\Rightarrow$ $$75x - 48x = 64 + 125$$

$\Rightarrow$ $$27x = 189$$

$\Rightarrow$ $$x = \frac{189}{27}$$

$\Rightarrow$ $$x = 7$$

Factor Theorem

[CHAPTER – 8]

1. Find the remainder in the following case, when $f(x)$ is divided by $g(x)$:
$f(x) = 2x^3 - 3x^2 - 4x - 5,\ g(x) = 2x + 1.$

Sol. Given, $f(x) = 2x^3 - 3x^2 - 4x - 5; g(x) = 2x + 1.$
By Remainder theorem, when $f(x)$ is divided by
$g(x) = 2x + 1$, then $g(x) = 0$

$$\Rightarrow \quad 2x + 1 = 0 \quad \Rightarrow x = -1/2$$

$$\text{Remainder} = f\left(-\frac{1}{2}\right)$$

$$= 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 5$$

$$= 2 \times \left(\frac{-1}{8}\right) - 3 \times \frac{1}{4} + \frac{4}{2} - 5 = -\frac{1}{4} - \frac{3}{4} + 2 - 5$$

$$= \frac{-1-3+8-20}{4} = \frac{-16}{4} = -4$$

2. The polynomial $p(x) = kx^3 + 9x^2 + 4x - 8$, when divided by the polynomial $q(x) = x + 3$, leaves the remainder (–20). Find the value of k.

Sol. Put $q(x) = 0 \Rightarrow x + 3 = 0 \Rightarrow x = -3$
Put $x = -3$ in $p(x) = kx^3 + 9x^2 + 4x - 8$, we get

$$\Rightarrow \quad k(-3)^3 + 9(-3)^2 + 4(-3) - 8 = -20$$
$$\Rightarrow \quad -27k + 81 - 12 - 8 = -20$$
$$\Rightarrow \quad -27k + 61 = -20$$
$$\Rightarrow \quad -27k = -81$$
$$\text{So,} \quad k = 3$$

3. Use the Remainder Theorem to factorise the following expression:
$2x^3 + x^2 - 13x + 6$

Sol. Polynomial is $2x^3 + x^2 - 13x + 6$
On putting $x = 2$, we get

$$2(2)^3 + (2)^2 - 13(2) + 6 = 2 \times 8 + 4 - 13 \times 2 + 6$$
$$= 16 + 4 - 26 + 6 = 26 - 26 = 0$$

So, $(x - 2)$ is a factor of given polynomial.
[Dividing polynomial $2x^3 + x^2 - 13x + 6$ by $(x - 2)$, we get

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & \underset{-}{2x^3} \underset{+}{-} 4x^2 \\ \hline & 5x^2 - 13x + 6 \\ & \underset{-}{5x^2} \underset{+}{-} 10x \\ \hline & -3x + 6 \\ & \underset{+}{-}3x \underset{-}{+} 6 \\ \hline & \times \end{array}$$

$$\text{Now,} \quad 2x^2 + 5x - 3 = 2x^2 + 6x - x - 3$$
$$= 2x(x + 3) - 1(x + 3)$$
$$= (2x - 1)(x + 3).$$

$$\text{So} \quad 2x^3 + x^2 - 13x + 6 = (x - 2)(2x - 1)(x + 3).$$

4. Using the Remainder Theorem, factorise completely the following polynomial:

$3x^3 + 2x^2 - 19x + 6$

Sol. Let $\quad p(x) = 3x^3 + 2x^2 - 19x + 6$

For $x = 2$, $\quad p(2) = 3(2)^3 + 2(2)^2 - 19 \times 2 + 6$

$= 24 + 8 - 38 + 6 = 38 - 38 = 0$

$\therefore$ $(x - 2)$ is a factor of $p(x)$.

$$
\begin{array}{r|l}
 & 3x^2 + 8x - 3 \\
\hline
x - 2 & 3x^3 + 2x^2 - 19x + 6 \\
 & 3x^3 - 6x^2 \\
 & \underline{-\quad +\qquad\qquad\qquad} \\
 & \qquad 8x^2 - 19x + 6 \\
 & \qquad 8x^2 - 16x \\
 & \qquad \underline{-\quad +\qquad\quad} \\
 & \qquad\qquad -3x + 6 \\
 & \qquad\qquad -3x + 6 \\
 & \qquad\qquad \underline{+\quad -} \\
 & \qquad\qquad\quad \times
\end{array}
$$

Now $\quad 3x^2 + 8x - 3 = 3x^2 + 9x - x - 3$

$= 3x(x + 3) - 1(x + 3)$

$= (x + 3)(3x - 1)$

$\therefore \quad 3x^3 + 2x^2 - 19x + 6 = (x - 2)(x + 3)(3x - 1)$

5. When divided by $x - 3$, the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'.

Sol. When the polynomial $x^3 - px^2 + x + 6$ is divided by $(x - 3)$, the remainder $= (3)^3 - p(3)^2 + 3 + 6$

$= 27 - 9p + 9 = 36 - 9p$

Also, when the polynomial $2x^3 - x^2 - (p + 3)x - 6$ is divided by $(x - 3)$,

the remainder $= 2(3)^3 - (3)^2 - (p + 3) \times 3 - 6$

$= 54 - 9 - 3p - 9 - 6 = 54 - 24 - 3p = 30 - 3p$

According to the question,

$\therefore \quad 30 - 3p = 36 - 9p \Rightarrow 9p - 3p = 36 - 30$

$\Rightarrow \quad 6p = 6 \Rightarrow p = 1$

6. Find the value of 'k' if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$. Hence, determine whether $(x + 5)$ is also a factor.

Sol. Since $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$,

Therefore, $\quad (2)^3 + 2(2)^2 - k(2) + 10 = 0$

or $\quad 8 + 8 - 2k + 10 = 0$

or $\quad 26 - 2k = 0$

or $\quad -2k = -26$

or $\quad k = 13$

Put $x + 5 = 0$, so $x = -5$

Putting $x = -5$ in the given polynomial, $x^3 + 2x^2 - 13x + 10$, we get the

Remainder

$= (-5)^3 + 2(-5)^2 - 13(-5) + 10$

$= -125 + 50 + 65 + 10 = -125 + 125 = 0$

So, $(x + 5)$ is also a factor of the given polynomial.

7. Show that $(x-1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the above expression.

Sol. For $x = 1$

$$\begin{aligned} f(x) &= x^3 - 7x^2 + 14x - 8 = (1)^3 - 7(1)^2 + 14(1) - 8 \\ &= 1 - 7 + 14 - 8 = 15 - 15 = 0 \end{aligned}$$

Hence $(x-1)$ is a factor of $x^3 - 7x^2 + 14x - 8$

Now,

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x - 1 & x^3 - 7x^2 + 14x - 8 \\ & x^3 - x^2 \\ & \underline{- \quad +} \\ & -6x^2 + 14x - 8 \\ & -6x^2 + 6x \\ & \underline{+ \quad -} \\ & 8x - 8 \end{array}$$

Also, $$\begin{aligned} x^2 - 6x + 8 &= x^2 - 4x - 2x + 8 \\ &= x(x-4) - 2(x-4) = (x-2)(x-4) \end{aligned}$$

Hence, $x^3 - 7x^2 + 14x - 8 = (x-1)(x-2)(x-4)$.

8. If $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$,

(i) find the value of p.

(ii) factorise the above expression completely.

Sol. (i) $\because$ $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$

$\therefore$ $2(2)^3 - (2)^2 - p(2) - 2 = 0$

$\Rightarrow$ $16 - 4 - 2p - 2 = 0$

$\Rightarrow$ $10 - 2p = 0$

or $10 = 2p \Rightarrow p = 5$

(ii) Given expression becomes $2x^3 - x^2 - 5x - 2$

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline x - 2 & 2x^3 - x^2 - 5x - 2 \\ & 2x^3 - 4x^2 \\ & \underline{- \quad +} \\ & 3x^2 - 5x - 2 \\ & 3x^2 - 6x \\ & \underline{- \quad +} \\ & x - 2 \\ & x - 2 \\ & \underline{- \quad +} \\ & \times \end{array}$$

$$\begin{aligned} 2x^2 + 3x + 1 &= 2x^2 + 2x + x + 1 \\ &= 2x(x+1) + 1(x+1) \\ &= (x+1)(2x+1) \end{aligned}$$

Thus, $2x^3 - x^2 - 5x - 2 = (x-2)(x+1)(2x+1)$

9. For what value of 'a' the polynomial $g(x) = 2x^2 - 3x$ is a factor of $f(x) = x^3 - 2x^2 + ax - 3a$?

Sol. Put $2x^2 - 3x = 0 \Rightarrow x(2x-3) = 0$

$\Rightarrow$ Either $x = 0$ or $x = \frac{3}{2}$

So, put $f(0) = 0$ and $f(3/2) = 0$

$$f(x) = x^3 - 2x^2 + ax - 3a$$

$$\Rightarrow \quad f(0) = (0)^3 - 2(0)^2 + a(0) - 3a = 0$$

$$\Rightarrow \quad -3a = 0 \Rightarrow a = 0$$

and $$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3 - 2\left(\frac{3}{2}\right)^2 + a\left(\frac{3}{2}\right) - 3a = 0$$

$$\Rightarrow \quad \frac{27}{8} - \frac{9}{2} + \frac{3a}{2} - 3a = 0 \Rightarrow \frac{27 - 36 + 12a - 24a}{8} = 0$$

$$\Rightarrow \quad -12a - 9 = 0 \Rightarrow -12a = 9$$

$$\Rightarrow \quad a = -\frac{3}{4}$$

Arithmetic Progression

[CHAPTER – 10]

Q.1. Show that the progression 11, 6, 1, − 4, −9,.... is an A.P. Find its first term and the common difference.

Ans. Clearly, (6 −11) = (1− 6) = (− 4 − 1)=(− 9+4) = -5 (constant)

Thus, each term differs from its preceding term by − 5.

So, the given progression is an A.P.

Its first term = 11 and common difference = −5.

Q.2. What is 18th term of the sequence defined by $a_n = \frac{n(n-3)}{n+4}$.

Ans. We have, $a_n = \frac{n(n-3)}{n+4}$,

Putting n=18, we get $a_{18} = \frac{18 \times (18-3)}{18+4} = \frac{18 \times 15}{22} = \frac{135}{11}$

Q.3. If seven times the seventh term of an A.P. is equal to eleven times its eleventh term, show that its eighteenth term is zero.

Ans. Let the first term be a and common difference be d. Then,

$$7\times t_7=11\times t_{11}\Rightarrow 7\times(a+6d)=11\times(a+10d)$$

$$\Rightarrow 7a+42d=11a+110d\Rightarrow 7a-11a=110d-42d$$

$$\Rightarrow -4a=68d\Rightarrow a=-17d \quad ...(1)$$

Now, $t_{18}=a+17d=-17d+17d=0$

Hence, eighteenth term is zero.

Q.4. If $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b. Then, find the value of n.

Ans. We have A.M. between a and b= $b=\dfrac{a+b}{2}$

It is given that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b.

$$\therefore \frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}\Rightarrow 2(a^{n+1}+b^{n+1})=(a^n+b^n)(a+b)$$

$$\Rightarrow 2a^{n+1}+2b^{n+1}=a^{n+1}+ab^n+a^nb+b^{n+1}\Rightarrow a^{n+1}+b^{n+1}=ab^n+a^nb$$

$$\Rightarrow a^{n+1}-a^nb=ab^n-a^{n+1}\Rightarrow a^n(a-b)=b^n(a-b)\Rightarrow a^n=b^n$$

$$\Rightarrow \frac{a^n}{b^n}=1\Rightarrow\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^0\Rightarrow n=0$$

Q.5. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Ans. The number of rose plants in the 1st, 2nd , 3rd , ... Last rows are :

23, 21, 19, ..., 5

It forms an A.P. Let the number of rows in the flower bed be n.

Then, $a=23,\ d=21-23=-2,\ t_n=5$

As, $t_n=a+(n-1)d\Rightarrow 5=23+(n-1)(-2)$

$$\Rightarrow -18=(n-1)(-2)\Rightarrow n=10$$

So, there are 10 rows in the flower bed.

Q.6. Find the sum of first 24 terms of the list of numbers whose n^{th} term is given by

$a_n = 3 + 2n.$

Ans. As $a_1 = 3 + 2n$

So, $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

....

....

List of numbers becomes 5, 7, 9, 11, ...

Here $7 - 5 = 9 - 7 = 11 - 9 = 2$ and so on. So it forms an A.P. with common difference $d = 2.$

To find S_{24}, We have $n = 24, a = 5, d = 2$

Therefore, $S_{24} = \frac{24}{2}[2 \times 5 + (24 - 1) \times 2] = 12[10 + 46] = 672$

So, sum of first 24 terms of the list of numbers is 672.

Q.7. If the p^{th}, q^{th} and n^{th} terms of an A.P. be a, b, c respectively, then show that

$a(q - r) + b(r - p) + c(p - q) = 0$

Ans. Let x be the first term and d be the common difference of the given A.P. Then

$t_p = x + (p - 1)d, \; t_q = x + (q - 1)d \; and \; t_r = x + (r - 1)d$

$\therefore \; x + (p - 1)d = a$ ...(1)

$x + (q - 1)d = b$ ...(2)

$x + (r - 1)d = c$ ...(3)

On multiplying (1) by $(q - r)$, (2) $by (r - p))$ and (3) $by (p - q)$ and adding, we get

$$a(q - r) + b(r - p) + c(p - q) = x(q - r + r - p + p - q)$$
$$+d[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$$

$= x\times 0+d\times 0$

Hence, $a(q-r)+b(r-p)+c(p-q)=0$. Proved.

Q.8. If a, b, c are in A.P., show that

i) $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ are in A.P. ii) $a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

Ans. i) Given a, b, c are in A.P. $\Rightarrow b-a=c-b$

Now, if $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ are in A.P., then $\frac{1}{ca}-\frac{1}{bc}=\frac{1}{ab}-\frac{1}{ca}$

$\Rightarrow \frac{b-a}{abc}=\frac{c-b}{abc}\Rightarrow b-a=c-b$

This is true from (1).

Hence, $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ are in A.P. Proved.

ii) Let $a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

$\Rightarrow a^2(b+c)+abc, b^2(c+a)+abc, c^2(a+b)+abc$ are in A.P.

[adding abc to each term]

$\Rightarrow a(ab+ac+bc), b(bc+ab+ac), c(ca+cb+ab)$ are in A.P.

$\Rightarrow a, b, c$ are in A.P. [Dividing each term by $(ab+bc+c)$]

This is given to be true.

$\therefore a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P. Proved.

Q.9. If the nth term of the A.P.9,7,5, ... is same as the nth term of the A.P.15,12,9,.... find n.

Ans. Given A.P.s are 9, 7, 5 ..... and 15, 12, 9 ......

Let a_1, d_1 and a_2, d_2 be the first terms and common difference of two A.P. respectively. So $a_1=9,\ d_1=-2, a_2=15, d_2=-3$

According to question nth term of two A.P. are same

$\Rightarrow a_1+(n-1)d_1=a_2+(n-1)d_2$

$\Rightarrow 9+(n-1)(-2)=15+(n-1)(-3)$

$\Rightarrow 3(n-1)-2(n-1)=15-9 \Rightarrow (x-1)=6 \Rightarrow n=7.$

Q.10. The 7^{th} term of the A.P. is 32 and its 13^{th} term is 62. Find the A.P.

Ans. Let a and d be first term and common difference respectively of the A.P.

So, $a_7 = 32 \Rightarrow a+6d=32$ ... (i)

$a_{13} = 62 \Rightarrow a+12d=62$ ...(ii)

Subtracting(i) from (ii), we get $6d=39 \Rightarrow d=5$

By (i) we get $a+30=32 \Rightarrow a=2$

So, A.P. is a, a+b, a+2d, a+3d....... or 2, 2+5, 2+10, 2+15......

Or 2, 7, 12, 17 ....

Q.11. The sum of three numbers in A.P. is 12 and the sum of their cubes is 188. Find the numbers.

Ans. Let three numbers in A.P. be $a-d, a, a+d$

So, $a-d+a+a+d=12 \Rightarrow 3a=12 \Rightarrow a=4$

Also $(a-d)^3+a^3+(a+d)^3=288$

$\Rightarrow (4-d)^3+4^3+(4+d)^3=288$

$\Rightarrow 64-d^3-48d+12d^2+64+64+d^3+48d+12d^2=288$

$\Rightarrow 192+24d^2=288 \Rightarrow 24d^2=96 \Rightarrow d^2=4$

$\therefore d^2=\pm 2$

So, numbers are $4-2, 4, 4+2$ *or* $4-(-2), 4, 4+(-2)$ *or* $2, 4$ *and* 6 *or* $6, 4$ *and* 2.

Q.12. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum or n terms.

Ans. Let a and d be respectively first term and common difference of given A.P. so we have

$S_7 = 49 \Rightarrow \frac{7}{2}(29+6d)=49$

$\Rightarrow 2a+6d=14$ ...(i)

And $S_7 = 289 \Rightarrow \frac{17}{2}(2a+16d)=289$

$\Rightarrow 2a+16d=34$

Subtracting (i) from (ii) we get

$10d=20 \Rightarrow d=2$ and by(i) $a=1$

So sum of first n term $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2.1+(n-1)2]$

$= \frac{n}{2}[2+2n-2] = \frac{n\times 2n}{2} = n^2$

Q.13. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. We have $a=5$ and $l=45$ and $sum=400$

$\Rightarrow \frac{n}{2}(a+l)=400 \Rightarrow \frac{n}{2}(5+45)=400$

$\Rightarrow n=\frac{400\times 2}{50}=16$

Also $a+(16-1)d=45 \Rightarrow 15d=45-5 \Rightarrow 15d=40$

$\Rightarrow d=\frac{40}{15}=\frac{8}{3}$

Therefore number of terms =16 and common difference $=\frac{8}{3}$

Q.14. In an A.P. the first term is 22, nth term is -11 and the sum to first n terms is 66. Find n and d, the common difference.

Ans. We have a=22, a_n=-11 and S_n=66

Now $a_n=-11 \Rightarrow a+(n-1)d=-11$

$\Rightarrow 22+(n-1)d=-11 \Rightarrow (n-1)d=-33$ ... (i)

and $S_n=66 \Rightarrow \frac{n}{2}[2a+(n-1)d]=66$

$\Rightarrow n[2\times 22+(n-1)d]=132$

$\Rightarrow n[44-33]=132$ [using (i)]

$\Rightarrow 11n=132 \Rightarrow n=12$

By (i) $11d=-33 \Rightarrow d=-3$

So number of terms =12 and common difference=-3.

Q.15. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. Here, $a=5, l=45=T_n, S_n=400$

$\because T_n = a+(n-1)d$

$\therefore 45=5+(n-1)d$

$\Rightarrow (n-1)d=45-5 \Rightarrow (n-1)d=40$ ...(1)

Also $S_n=\frac{n}{2}(a+l)$

$\Rightarrow 400=\frac{n}{2}(5+45) \Rightarrow 400\times 2=n\times 50$

$\Rightarrow n=\frac{400\times 2}{50}=16$

From (1), we get $(16-1)d=40 \Rightarrow 15d=40 \Rightarrow d=\frac{8}{3}$

Q.16. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. Here, $n=51, T_2=14$ *and* $T_3=18$

Let the first term of the A.P. be a and the common difference is d.

We have $T_2=a+d \Rightarrow a+d=14$ .... (1)

$T_3=a+2d \Rightarrow a+2d=18$ .... (2)

Subtracting(1) from (2), we get

$a+2d-a-d=18-14 \Rightarrow d=14$

From (1), we get $a+d=14 \Rightarrow a+4=14 \Rightarrow a=14-4=10$

Now, $S_n = \frac{n}{2}[(2a+(n-1)d]$

$\Rightarrow S_{51} = \frac{51}{2}[(2\times10)+(51-1)\times4]$

$= \frac{51}{2}[20+200] = \frac{51}{2}[220] = 51\times110 = 5610$

Thus, the sum of 51 terms is 5610.

Q.17. If the sum of first 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans. Here, we have $S_7 = 49$ and $S_{17} = 289$

Let the first term of the A.P. be 'a' and 'd' be the common difference, then

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2a+(7-1)d] = 49$

$\Rightarrow 7(2a+6d) = 2\times49 = 98$

$\Rightarrow 2a+6d = \frac{98}{7} = 14 \Rightarrow 2[a+3d] = 14$

$\Rightarrow a+3d = \frac{14}{2} = 7 \Rightarrow a+3d = 7$ .... (1)

Also, $S_{17} = \frac{17}{2}[2a+(17-1)d] = 289$

$\Rightarrow \frac{17}{2}(2a+16d) = 289$

$\Rightarrow a+8d = \frac{289}{17} = 17 \Rightarrow a+8d = 17$ .... (2)

Subtracting (1) from (2), we have

$a+8d-a-3d = 17-7 \Rightarrow 5d = 10 \Rightarrow d = 2$

Now, from (1), we have

$a+3(2) = 7 \Rightarrow a = 7-6 = 1$

Now, $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[2\times1+(n-1)\times2]$

$= \frac{n}{2}[2+2n-2] = \frac{n}{2}[2n] = n\times n = n^2$

Thus, the required sum of n terms = n^2.

Q.18. Show that $a_1, a_2 ....., a_n, ...$ from an A.P. where a_n is defined as below:

i) $a_n = 3+4_n$ (ii) $a_n = 9-5n$

Also find the sum of the first 15 terms in each case.

Ans. i) Here, $a_n = 3+4_n$

Putting $n = 1, 2, 3, 4, ..., n,$ we get

$a_1 = 3+4(1) = 7$

$a_2 = 3+4(2) = 11$

$a_3 = 3+4(3) = 15$

$a_4 = 3+4(4) = 19$

....

$a_n = 3+4n$

$\therefore$ The A.P. in which $a = 7$ and $d = 11-7 = 4$ $is 7, 11, 15, 19, ...(3+4n)$.

Now, $S_{15} = \frac{15}{2}[(2\times7)+(15-1)\times4]$

$= \frac{15}{2}[14+(14\times4)] = \frac{15}{2}[14+56] = \frac{15}{2}[70]$

$= 15\times35 = 525$

ii) Here, $a_n = 9-5n$

Putting $n = 1, 2, 3, 4, ..., n,$ we get

$a_1 = 9-5(1) = 4$

$a_2 = 9-5(2) = -1$

$a_3 = 9-5(3) = -6$

$a_4 = 9-5(4) = -11$

....

$a_n = 9-5x$

$\therefore$ The A.P. is $4, -1, -6, -11, ... 9-5n$ [having first term as 4 and $d = -1-4 = -5$]

$\therefore S_{15} = \frac{15}{2}[(2\times4)+(15-1)\times(-5)]$

$= \frac{15}{2}[8+14\times(-5)] = \frac{15}{2}[8-70] = \frac{15}{2}\times(-62)$

$= 15\times(-31) = -465.$

Q.19. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, ... (6 x 40)

And, these numbers are in A.P., such that a=6

$d = 12-6 = 6$ and $a_n = 6\times40 = 240 = l$

$\therefore S_{40} = \frac{40}{2}[(2\times6)+(40-1)\times6]$

$= 20[12+39\times6] = 20[12+234] = 20\times246 = 4920$

OR

$S_n = \frac{n}{2}[a+l] \Rightarrow S_{40} = \frac{40}{2}[6+240] = 20\times246 = 4920$

Thus, the sum of first 40 multiples of 6 is 4920.

Q.20. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are

$8, (8\times2), (8\times3), (8\times4), ..., (8\times15)$ or $8, 16, 24, 32, ..., 120$

These number are in A.P., where $a = 8$ and $l = 120$

$\therefore S_{15} = \frac{15}{2}[a+l] = \frac{15}{2}[8+120] = \frac{15}{2}\times128 = 15\times64 = 960$

Thus, the sum of first 15 multiples of 8 is 960.

Reflection
[CHAPTER – 12]

1. Using a graph paper, plot the points A(6, 4) and B(0, 4).

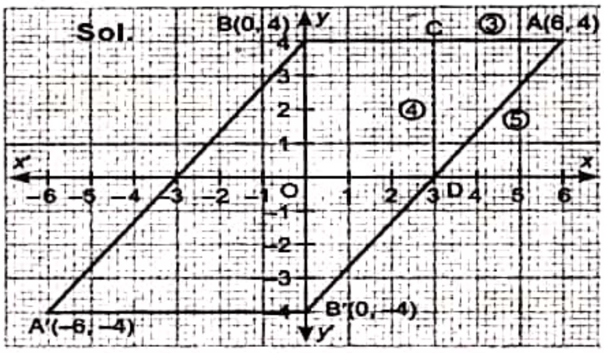
(i) Reflect A and B in the origin to get the images A′ and B′.

(ii) Write the coordinates of A′ and B′.

(iii) State the geometrical name for the figure ABA′B′.

(iv) Find its perimeter.

Sol. (i)



(ii) $A' = (-6, -4)$

$B' = (0, -4)$

(iii) Parallelogram

(iv)
$$AD^2 = AC^2 + CD^2$$
$$= 3^2 + 4^2$$
$$= 9 + 16 = 25$$
$$AD = 5 \text{ units}$$
$$\text{Perimeter} = AB + BA' + A'B' + B'A$$
$$= 6 + 10 + 6 + 10$$
$$= 32 \text{ sq. units}$$

2. Use graph paper to answer the following questions: (Take 2 cm = 1 unit on both axes)

(i) Plot the points A(−4, 2) and B(2, 4)

(ii) A′ is the image of A when reflected in the y-axis. Plot it on the graph paper and write the coordinates of A′.

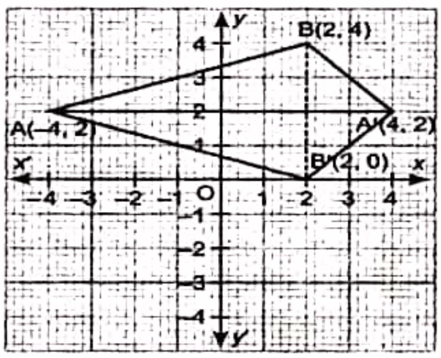
(iii) B′ is the image of B when reflected in the line AA′. Write the coordinate of B′.

(iv) Write the geometric name of the figure ABA′B′.

(v) Name a line of symmetry of the figure formed.

Sol.

(i)



(ii) $A' = (4, 2)$

(iii) $B' = (2, 0)$

(iv) Kite

(v) Line of symmetry = AA′.

Section Formula

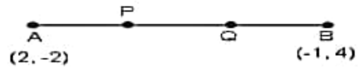
[CHAPTER – 13]

Q.1. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, -2) and B(-7, 4).

Ans. Let P and Q be the points of trisection of AB, i.e., AP=PQ=QB.

Therefore, P divides AB in the ratio 1:2. Hence, the coordinates of P are

$$\left(\frac{1\times(-7)+2\times 2}{1+2}, \frac{1\times 4+2\times(-2)}{1+2}\right). \text{ i.e., } (-1, 0)$$



Now, Q divides AB in the ratio 2:1. So, the coordinates of Q are

$$\left(\frac{2\times(-7)+1\times 2}{2+1}, \frac{2\times 4+1\times(-2)}{2+1}\right), i.e., (-4, 2)$$

Therefore, the coordinates of the points of trisection of the line segment joining

A and B are $(-1, 0)$ and $(-4, 2)$

Q.2. Find the centroid of the triangle formed by the vertices (2, 3), (-2, -5) and (-4, 6).

Ans. The coordinates of the centroid of the triangle formed by given vertices are

$$\left(\frac{2+(-2)+(-4)}{3}, \frac{3+(-5)+6}{3}\right), i.e., \left(-\frac{4}{3}, \frac{4}{3}\right)$$

Q.3. The coordinates of the centroid of a triangle are (1,3) and two of its vertices are

(-7, 6) and (   ). Find the third vertex of the triangle.

Ans. Let the third vertex of the triangle be (p,q). Then

$$1 = \frac{-7+8+p}{3} \text{ and } 3 = \frac{6+5+q}{3}$$

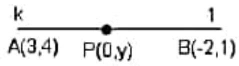
$$\Rightarrow p = 2 \text{ and } q = -2$$

Thus, the coordinates of the third vertex are (2, -2).

Q.4. Find the ratio in which the line segment joining A(3, 4) and B(-2, 1) is divided by the y – axis.

Ans. Let the line segment AB cut (or meet) the y-axis at the point P.

Then, the coordinates of P are (0, y).



Let P divide AB in the ratio k:1.

Then, $0 = \frac{-2k+3}{k+1} \Rightarrow k = \frac{3}{2}$

Hence, the required ration is $\frac{3}{2}:1 \ or \ 3:2$.

Q.5. Find the mid-point of the line segment AB, joining the points $A(-8,6)$ and $B(12,-2)$.

Ans. Let $P(x, y)$ be the mid-point of AB. Then,

$$x = \frac{-8+12}{2},\ y = \frac{6+(-2)}{2}$$

$\Rightarrow x = 2$ and $y = 2$

Thus, the mid-point of AB is (2,2)

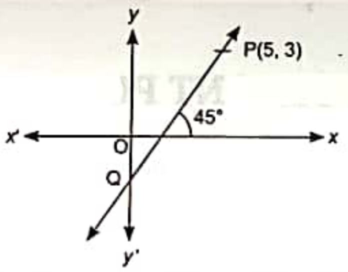
Equation of a line
[CHAPTER – 14]

Ex.1. The line through P(5, 3) intersects y-axis at Q.

(*i*) Write the slope of the line.

(*ii*) Write the equation of the line.

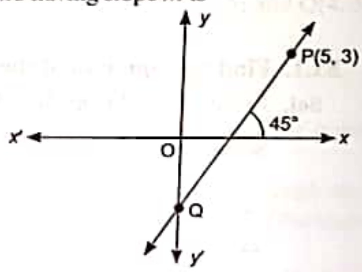
(*iii*) Find the coordinates of Q.



Sol. (*i*) Slope (m) of the line PQ = $\tan\theta = \tan 45° = 1$

(*ii*) From formula, the equation of the line passing through point (x_1, y_1) and having slope m is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y - 3 = 1(x - 5)$$
$$\Rightarrow \quad y - 3 = x - 5$$
$$\Rightarrow \quad x - y - 2 = 0$$



(*iii*) Let the coordinates of Q = $(0, a)$

It lies on the line PQ, i.e.

$$x - y - 2 = 0$$

So $\quad 0 - a - 2 = 0$

$\Rightarrow \quad a = -2$

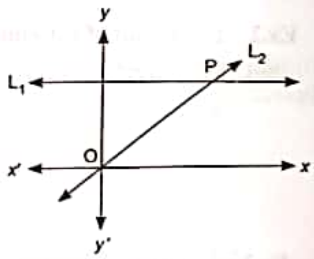
So, coordinates of Q = $(0, -2)$

Ex.2. Given equation of line L_1 is $y = 4$.

(*i*) Write the slope of line L_2 if L_2 is the bisector of angle O.

(*ii*) Write the co-ordinates of point P.

(*iii*) Find the equation of L_2. **[2011]**



Sol. Given : L_1 is $y = 4$

(*i*) If L_2 is the bisector of angle O, then L_2 makes 45° angle with x-axis.

Hence slope (m) = $\tan\theta = \tan 45° = 1$

(*ii*) PR = 4 $\quad (\because L_1$ is $y = 4)$

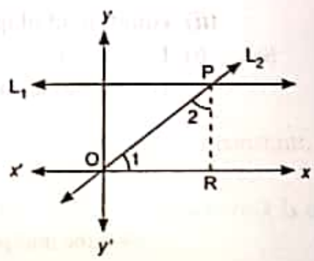
$\angle 1 = 45°$

So, $\angle 2 = 45°$

Hence OR = PR

$\Rightarrow$ OR = 4

$\therefore$ Coordinates of point P = (4, 4)



(*iii*) Equation of line L_2 is

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y - 0 = 1(x - 0)$$
$$\Rightarrow \quad y = x$$

Q 3. Find the equation of the straight line passing through the pair of points (a, b) and $(a+b, a-b)$.

Sol. Points are (a, b) and $(a+b, a-b)$

So, equation of line be, $y - b = \frac{a-b-b}{a+b-a}(x-a) \Rightarrow y - b = \frac{a-2b}{b}(x-a)$,

$$\Rightarrow \quad b(y-b) = (a-2b)(x-a)$$
$$\Rightarrow \quad by - b^2 = ax - a^2 - 2bx + 2ab$$
$$\Rightarrow \quad by - b^2 - ax + a^2 + 2bx - 2ab = 0$$
$$\Rightarrow \quad (2b-a)x + by + (a^2 - b^2 - 2ab) = 0$$

Q 4. The graph of the equation $ax + by + 5 = 0$ passes through the points (1, 2) and (–1, 0). Find a and b.

Sol. Since graph of the equation $ax + by + 5 = 0$ passes through the points (1, 2) and (–1, 0).

So, $1a + 2b + 5 = 0$ and $-a + 5 = 0$

$\Rightarrow$ $a + 2b + 5 = 0$ and $-a + 5 = 0$ so, $a = 5$

$\Rightarrow$ $5 + 2b + 5 = 0$

$\Rightarrow$ $2b + 10 = 0 \Rightarrow b = -5$

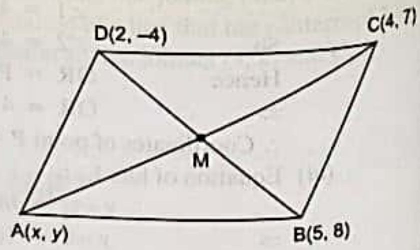
Q 5. ABCD is a parallelogram where A (x, y), B (5, 8), C (4, 7) and D (2, – 4). Find

(i) coordinates of A.

(ii) equation of diagonal BD.

Sol. (i) Let M be the point of intersection of the diagonals AC and BD.

Let the coordinate of M be (a, b).



So, $a = \frac{5+2}{2}, b = \frac{-4+8}{2}$

$\Rightarrow$ $a = \frac{7}{2}, b = 2$

$\because$ M is the mid-point of AC,

So $\frac{7}{2} = \frac{x+4}{2}, 2 = \frac{y+7}{2}$

$\Rightarrow$ $x = 3, y = -3$

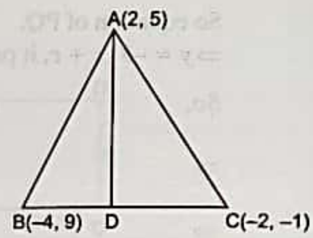
So, the coordinates are A are (3, –3).

(ii) Equation of diagonal BD, $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow$ $y - 8 = \frac{-4-8}{2-5}(x-5)$

$$\Rightarrow \quad y - 8 = \frac{-12}{-3}(x-5)$$
$$\Rightarrow \quad y - 8 = 4(x-5)$$
$$\Rightarrow \quad y - 8 = 4x - 20$$
$$\Rightarrow \quad 4x - y - 12 = 0$$

Q 6. Find the equation of the median AD of the ΔABC whose vertices are A(2, 5), B(–4, 9) and C(–2, –1).



Sol. Coordiantes of D $\equiv \left(\frac{-2-4}{2}, \frac{-1+9}{2}\right) \equiv (-3, 4)$

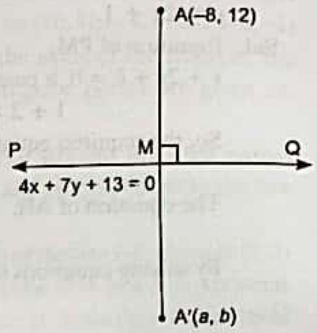
So the equation of AD,

$$y-5 = \frac{4-5}{-3-2}(x-2)$$

$$\Rightarrow \quad y-5 = \frac{1}{5}(x-2)$$

$\Rightarrow \quad 5y-25 = x-2 \Rightarrow x-5y+23 = 0$

Q 7. Find the image of the point (–8, 12) with respect to the line mirror $4x + 7y + 13 = 0$.



Sol. Coordinates of M $\equiv \left(\frac{a-8}{2}, \frac{b+12}{2}\right)$

It lies on $4x + 7y + 13 = 0$

$$\therefore \quad 4\left(\frac{a-8}{2}\right)+7\left(\frac{b+12}{2}\right)+13 = 0$$

$$\Rightarrow \quad 4a+7b+78 = 0 \qquad \text{...(i)}$$

Since $AA' \perp PQ$, so

$$\text{Slope (PQ)} \times \text{slope (AA')} = -1$$

$$\left(\frac{-4}{7}\right)\times\left(\frac{b-12}{a+8}\right) = -1$$

$$\Rightarrow \quad 7a-4b+104 = 0 \qquad \text{...(ii)}$$

Now multiplying eq (i) by 4 and eq (ii) by 7 and adding eq (i) and (ii), we get

$$16a + 28b = -312$$
$$49a - 28b = -728$$
$$65a = -1040$$

$$\Rightarrow \quad a = \frac{-1040}{65} = -16,$$

Put $a = -16$ in equation (i), we get

$$4a + 7b + 78 = 0$$
$$\Rightarrow \quad 4 \times (-16) + 7b + 78 = 0$$
$$\Rightarrow \quad -64 + 7b + 78 = 0$$
$$\Rightarrow \quad 7b + 14 = 0$$
$$\Rightarrow \quad 7b = -14 \Rightarrow b = -2$$

Image = (– 16, – 2).

Q.8. Show that the line joining (2, -3) and (-5,1) is

i) parallel to the line joining (7,-1) and (0, 3)

ii) perpendicular to the line joining (4,5) and (0, -2).

Ans. We know that the slope (m) of a line joining the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let l_1 be the line joining (2,-3) and (-5, 1) Then, Slope (m_1) of $l_1 = \frac{1+3}{-5-2} = -\frac{4}{7}$

i) Let l_2 be the line joining (7,-1) and (0,3). Then

slope (m_2) of $l_2 = \frac{3+1}{0-7} = -\frac{4}{7}$

since $m_1 = m_2$, the lines l_1 and l_2 are parallel.

ii) Let l_3 be the line joining (4, 5) and (0,-2). Then,

slope (m_3) of $l_3 = \frac{-2-5}{0-4} = \frac{7}{4}$

since $m_1 . m_3 = -\frac{4}{7} \times \frac{7}{4} = -1$, the lines l_1 and l_2 are perpendicular.

Q.9. Find the slope and the intercept on the y-axis of the following lines:

i) $3y + 2x = 12$ ii) $\frac{x+4}{2} - \frac{2(y-6)}{5} = 5$

Ans. i) $3y + 2x = 12$ gives

$$y = \frac{12-2x}{3}$$

$$\Rightarrow y = -\frac{2}{3}x + 4$$

So, the slope is $-\frac{2}{3}$ and the y-intercept is 4.

ii) $\frac{x+4}{2} - \frac{2(y-6)}{5} = 5$,gives

$$5(x+4) - 4(y-6) = 50 \Rightarrow 5x + 20 - 4y + 24 = 50$$

$$\Rightarrow 4y = 5x - 6$$

$$\Rightarrow y = \frac{5}{4}x - \frac{3}{2}$$

So, the slope is $\frac{5}{4}$ and the y-intercept is $-\frac{3}{2}$.

Q.10. Show that the points A(2,3) B(-1, -2) and C(5, 8) are collinear.

Ans. Slope of AB = $\frac{-2-3}{-1-2} = \frac{5}{3}$

Slope of BC= $\frac{8+2}{5+1} = \frac{10}{6} = \frac{5}{3}$.

Since slope of AB = Slope of BC, this means AB is parallel to BC

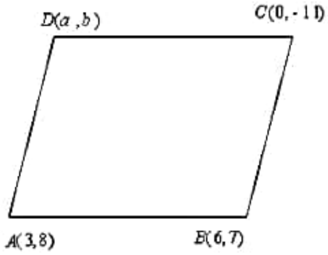
But B is a point common to both of them.

So, A, B and C are collinear.

Q.11. A(3,8), B(6, 7), C(0,-11) are three vertices of a // gm ABCD. Find the coordinates of the fourth vertex D, using slopes of the sides.

Ans. Let the fourth vertex D be D (α, β) .Then,

Slope of AB = $\frac{7-8}{6-3} = \frac{1}{3}$



Slope of BC = $\frac{-11-7}{0-6} = 3$

Slope of CD = $\frac{\beta-11}{\alpha-0} = \frac{\beta+11}{\alpha}$

Slope of DA= $\frac{8-\beta}{3-\alpha}$

Since ABCD is a $\| gm, AB \| CD$; and $BC \| DA$

Slope of AB = Slope of CD; and

Slope of BC = Slope of DA

$\Rightarrow -\frac{1}{3} = \frac{\beta+11}{a}$ and $3\frac{8-\beta}{3-\alpha}$

$\Rightarrow \alpha+3\beta+33=0$ and $3\alpha-\beta-1=0$

$\Rightarrow \alpha=-3$ and $\beta=-10$

Thus, the coordinates of the fourth vertex are (-3, -10)

Q.12. Find the equation of a straight line inclined at 45^0 with the positive direction of the x-axis and cutting off an intercept 3 on the negative side of the y-axis.

Ans. Here, $m = \tan 45^\circ = 1$ and $c=-3$

So, the equation of the straight line is $y = x-3$.

Q.13. Find the equation of a line passing through the point (1, 2) and inclined at 30^0 with the positive direction of the x-axis.

Ans. Here, $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$.

So, the equation of the straight line is $y = \frac{1}{\sqrt{3}} x + c$.

Since the line passes through the point (1,2), we have $2 = \frac{1}{\sqrt{3}} + c$

$\Rightarrow c = 2 - \frac{1}{\sqrt{3}}$

Hence, the required equation is $y = \frac{1}{\sqrt{3}} x + \left(2 - \frac{1}{\sqrt{3}}\right)$.

Q.14. The equation of a line is $y = 3x - 5$. Write down the slope of this line and the intercept made by it on the y-axis. Hence, or otherwise, write down the equation of the line which is parallel to this line and which passes through the point (0,5).

Ans. Here, $y = 3x - 5$

So, the slope of the line is 3 and the y-intercept is -5.

Now, the equation of the line parallel to $y = 3x - 5$ and passing through (0,5) is

$y - 5 = 3(x - 0)$

$\Rightarrow y = 3x + 5$

Q.15. Find the equation of a line passing through (2, -2) and is perpendicular to the line joining(-3,1) and (1,-2).

Ans. The slope of the line joining (-3,1) and (1,-2) is $\frac{-2-1}{1+3}$, i.e. $\frac{-3}{4}$.

So, the slope of the required line is $\frac{4}{3}$.

Hence, the required equation is $y + 2 = \frac{4}{3}(x - 2)$

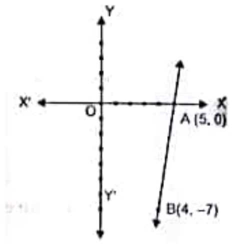
$\Rightarrow 3y = 4x - 14$

Q.16. In what ratio is the line joining the points (2,1) and (5,2) divided by the line joining (3,1) and (5,0)

Ans. The equation of the line joining the points A(3,2) and B(5,0) is

$$y-2=\frac{0-2}{5-3}(x-3)$$

i.e, $x+y-5=0$



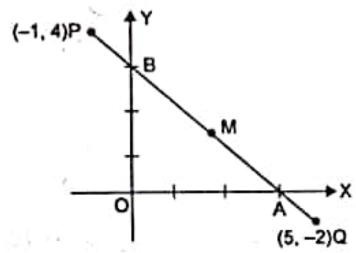
If (1) meets the line joining P(2,1) and Q(5,2) in M, then M divides PQ in the ratio k:1.

Then, the coordinates of M are $\left(\frac{5k+2}{k+1}, \frac{2k+1}{k+1}\right)$

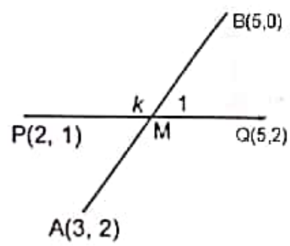
As M lies on (1), we have $\frac{5k+2}{k+1}+\frac{2k+1}{k+1}-5=0$

$$\Rightarrow 7k+3=5k-5=0$$

$$\Rightarrow 2k=2 \Rightarrow k=1$$



Thus, the required ratio is 1:1 i.e., M bisects PQ.



Q.17. Find the equation of a line with x-intercept =5. It means the line passes

through A(5,0)

The equation of the line passing through A(5,0) and B(4,-7) is

$$y-0=\frac{-7-0}{4-5}(x-5)$$

$$\Rightarrow y=7(x-5)$$

$$\Rightarrow y-7x+35=0$$

Q.18. A straight line posses through the points P(-1, 4) and Q(5,-2). If intersects the coordinate axes at points A and B. M is the mid-point of the segment AB. Find:

i) the equation of the line;

ii) the coordinates of A and B;

iii) the coordinate of M

Ans. i) Equation of the line PQ is $y-4=\frac{-2-4}{5+1}(x+1)$

i.e. $y-4=-(x+1)$

i.e., $x+y=3$

ii) Substituting y=0 in (1), we get x=3 and hence A(3,0)
Substituting x=0 in(1), we get y=3 and hence B(0,3)

Thus, the coordinates of A and B are (3,0) and (0,3) respectively.

iii) $M\left(\frac{3+0}{2},\frac{0+3}{2}\right), i.e., M\left(\frac{3}{2},\frac{3}{2}\right).$

Q.19. The coordinates of the vertex A of a square ABCD is (1,2) and the equation of the diagonal BD is $x+2y=10$ Find:

i) the equation of the diagonal AC

ii) the coordinates of the centre of the square.

iii) the coordinates of the intercepts made by the diagonal AC on the coordinate axes.

Ans. i) The diagonal AC is the line perpendicular to $x+2y=10$ and passing through A(1,2)

Slope of $x+2y=10$ is $-\frac{1}{2} \Rightarrow$ Slope of diagonal AC is 2.

$\therefore$ Equation of the diagonal AC is $y-2=2(x-1) \Rightarrow y=2x$

ii) Since the centre of the square is the intersection of the two diagonals AC and BD, we get the coordinates of the centre on solving the equations of AC and BD simultaneously, i.e., $y=2x$ and $x+2y=10$.
One solving, we get $x=2$ and $y=4$

$\therefore$ The centre of the square is (2, 4)

iii) The intercept made by the diagonal AC on the x-axis is 0 and on the y-axis is 0. Clearly, the diagonals passes through the origin.

Q.20. A(1,4), B(3, 2) and C(7,5) are the vertices of a ΔABC. Find:

i) the coordinates of the centroid G of ΔABC.

ii) the equation of a line, through G and parallel to AB.

Ans. i) Here, the coordinate of the centroid are:

$$G = \left(\frac{1+3+7}{3}, \frac{4+2+5}{3}\right) = \left(\frac{11}{3}, \frac{11}{3}\right).$$

ii) The required equations is

$$\Rightarrow y - \frac{11}{3} = \frac{2-4}{3-1}\left(x - \frac{11}{3}\right)$$

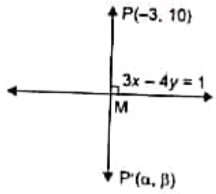
$$\Rightarrow y - \frac{11}{3} = -\left(x - \frac{11}{3}\right)$$

$$\Rightarrow y + x = \frac{22}{3}.$$

Q.21. i) Find the image of the point P(-3,10) in the line $3x-4y=1$

ii) Find the reflection of the point R(a, b) in the line $y = x$.

iii) A, B, C are the points (-1, 2) (4, 6), (8, 2) respectively. If AM is the median of ΔABC, find the coordinates of the image M' of M under reflection in the x-axis. Hence find the equation of the line AM.



Ans. i) Let the image of P(-3, 10) in the $3x-4y=1$ be P' (α, β).

Then M is the mid-point of PP' and M lies on the line l.

$$\therefore M\left(\frac{\alpha-3}{2}, \frac{\beta-10}{2}\right) = 1$$

and $3\left(\frac{\alpha-10}{2}\right) - 4\left(\frac{\beta+10}{2}\right) = 1$

$\Rightarrow 3\alpha - 4\beta - 51 = 0$ .... (2)

Also, $\frac{\beta-10}{\alpha+3} \times \frac{3}{4} = -1$

$\Rightarrow 4\alpha + 3\beta - 18 = 0$ .... (3)

Solving (2), we get P' (9, -6).

ii) Use the argument in Part(i) to prove the reflection A' of A in $y = x$ is (b,a)

iii) Since AM is a median of ΔABC, M is the mid-point of BC.

Coordinates of M are $\left(\frac{4+8}{2}, \frac{6+2}{2}\right) = (6, 4)$

Since M' is the reflection of M in x-axis, coordinates of M' are (6, -4).

AM' is a straight line with A(-1, 2) and M'(6,-4)

$\therefore$ Equation of AM′ is $y-2=\frac{-4-2}{6+1}(x+1)$

$\Rightarrow y-2=\frac{-6}{7}(x+1) \quad \Rightarrow 7y-14=-6x-6$

$\Rightarrow 6x+7y=8.$

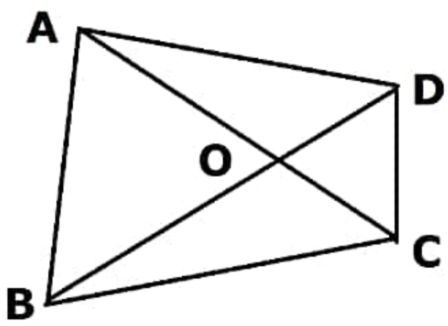
Similarity

[CHAPTER – 15]

1. In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O. If AO = 2CO and BO = 2DO; show that:

(i) ΔAOB is similar to ΔCOD.

(ii) OA*OD = OB*OC.



Ans. (i) In ΔAOB and ΔCOD,

$$\frac{AO}{CO}=2$$

$$\frac{BO}{DO}=2$$

$\frac{AO}{CO}=\frac{BO}{DO}$ [1]

$\angle$ AOB =$\angle$ COD(vertically opposite angles) [2]

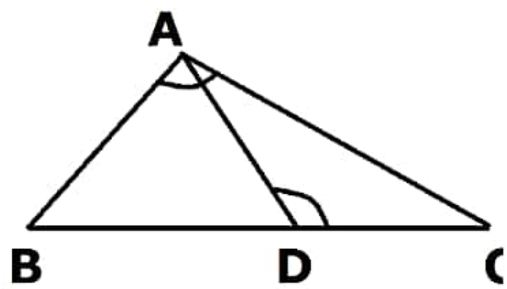
From equation 1 and 2,

$\Delta AOB \sim \Delta COD$(by S.A.S.)

(ii) From equation 1,

OA*OD = OB*OC [Proved]

2. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that: CA^2 = CB*CD.



Ans. In ΔABC & ΔDAC,

$\angle ACB = \angle DCA$(COMMON)

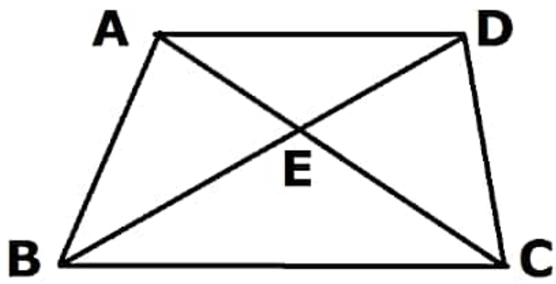
$\angle BAC = \angle ADC$

$\Delta ABC \sim \Delta DAC$(BY A.A.)

$\frac{CA}{CD} = \frac{CB}{CA}$ [CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION]

$CA^2 = CB*CD$ [PROVED]

3. In quadrilateral ABCD, diagonals AC and BD intersect at point E such that AE:EC = BE:ED. Show that ABCD is a trapezium.



Ans. In ΔAEB & ΔCED,

$\frac{AE}{EC} = \frac{BE}{ED}$(GIVEN)

$\angle AEB = \angle CED$ [VERTICALLY OPPOSITE ANGLES]

$\Delta AEB \sim \Delta CED$(BY S.A.S.)

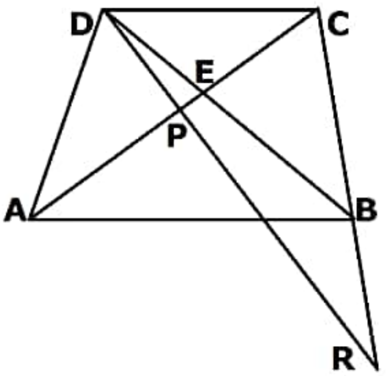
$\angle BAE = \angle DCE$ [CORRESPONDING ANGLES OF SIMILAR TRIANGLES ARE EQUAL]

Interior alternate angles are equal.

$AB \parallel DC$.

ABCD is a trapezium.

4. Given : ABCD is a rhombus, DPR and CBR are straight lines.



Ans. $\angle DPA = \angle RPC$(VERTICALLY OPP. ANGLES)

$\angle DAP = \angle RCP$(INTERIOR ALTERNATE ANGLES)

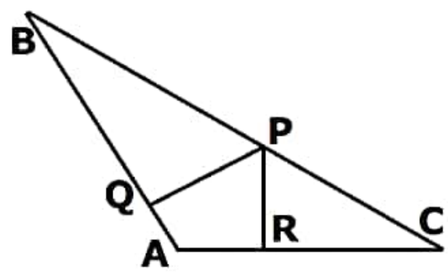
$\Delta DPA \sim \Delta RPC$(BY A.A.)

$\frac{DP}{PR} = \frac{AD}{CR}$ [CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION]

$\frac{DP}{PR} = \frac{DC}{CR}$ [AD=DC , SIDES OF A SQUARE ARE EQUAL]

DP*CR = DC*PR [PROVED]

5. Angle BAC of triangle ABC is obtuse and AB = AC. P is a point in BC such that pc = 12 cm. PQ and PR are perpendiculars to sides AB and AC respectively. If PQ = 15 cm and PR = 9 cm. find the length of PB.



Ans. In ΔBPQ & ΔCPR ,

$\angle PBQ = \angle PCR$ (GIVEN)

$\angle PQB = \angle PRC = 90°$(GIVEN)

$\Delta BPQ \sim \Delta CPR$(BY A.A.)

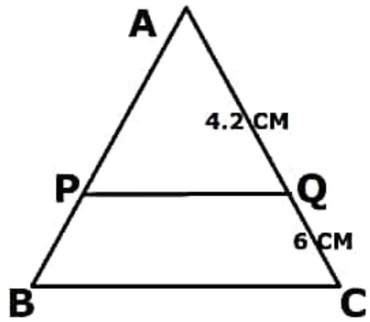
$\frac{BP}{CP} = \frac{PQ}{PR}$[CORRESPONDING SIDES OF SIMILAR

TRIANGLES ARE IN PROPORTION]

$$\frac{BP}{12} = \frac{15}{9}$$

$\therefore BP = 20$ cm

6. A line PQ is drawn parallel to the side BC of ΔABC which cuts side AB at P and side AC at Q. If AB = 9 cm, CA = 6 cm and AQ = 4.2 cm, find the length of AP.



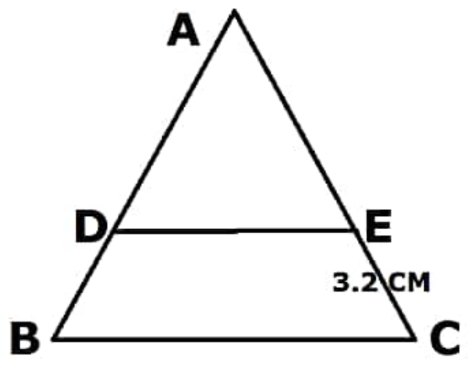
Ans. Since $\Delta APQ \sim \Delta ABC$(BY A.A.),

$$\frac{AP}{AB} = \frac{AQ}{AC}$$

$$\frac{AP}{9} = \frac{4.2}{6}$$

AP = 6.3 cm

7. A line segment DE is drawn parallel to base BC of ΔABC which cuts AB at point D and AC at point E. If AB = 5BD and EC = 3.2 cm, find the length of AE.



Ans. AB = 5BD

$$\frac{BD}{AB} = \frac{1}{5}$$

Since $\Delta ADE \sim \Delta ABC$(BY A.A.),

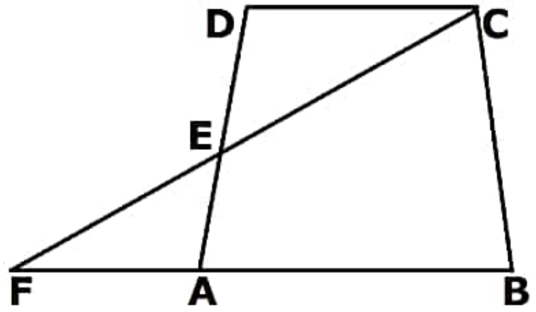
$$\frac{CE}{AC} = \frac{BD}{AB} = \frac{1}{5}$$

$$\frac{3.2}{AC} = \frac{1}{5}$$

AC = 16 cm

$\therefore$AE = AC − CE = 16 − 3.2 = 12.8 cm

8.Find the perimeter of the given parallelogram such that AE = 4 cm, AF = 8 cm, A B =12 cm, BF = 8+12 = 20 cm.



Ans. AD‖ BC

AE‖ BC

Since ΔFBC~ΔFAE(BY A.A.),

$$\frac{BF}{AF}=\frac{BC}{AE}$$

$$\frac{20}{8}=\frac{BC}{4}$$

BC = 10 cm

∴Perimeter of parallelogram ABCD = 2(l+b) = 2(AB+BC) = 2(12+10) = 2   22 = 44 cm

9. A model of a ship is made to a scale of 1:200. If the length of the model is 4 m; calculate the length of the ship.

Ans. Scale factor =k = 1/200

$$\frac{\text{length of the model}}{\text{length of ship}}=\frac{1}{200}$$

Length of ship = 200   length of the model = 200 X 4 = 800 m

10. A triangle ABC has been enlarged by scale factor m = 2.5 to the nearest A'B'C'. Calculate:

(i) the length of AB, if A'B' = 6 cm.

(ii) the length of C'A', if CA = 4 cm.

Ans. Scale factor =m= 2.5

(i) A'B' = m   AB

6 cm = 2.5   AB

AB = 2.4 cm

(ii) C'A' =m   CA

= 2.5   4 = 10 cm

11. A triangle LMN has been reduced by scale factor 0.8 to the triangle L'M'N'. Calculate:

(i) the length of M'N', if MN = 8 cm.

(ii) the length of LM, if L'M' = 5.4 cm.

Ans. Scale factor = m= 0.8

(i) M'N' = MN   m = 8   0.8 = 6.4 cm

(ii) L'M' = LM   m

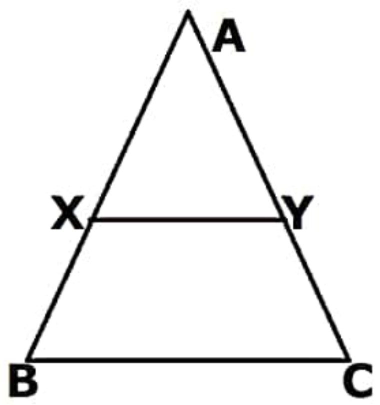
5.4 cm = LM   0.8

LM = 6.75 cm

12.

In the Following Figure,   is isosceles.
<AXY = <AYX.

If BX/AX = CY/AY,
show that triangle ABC



Ans. $\angle AXY = \angle AYX$

$$\frac{BX}{AX} = \frac{CY}{AY}$$

$$\frac{AX}{BX} = \frac{AY}{CY}$$

XY||BC [BY CONVERSE OF BASIC PROPORTIONALITY THEOREM]

$\angle AXY = \angle ABC$ ▮CORRESPONDING ANGLES

$\angle AYX = \angle ACB$ ▮CORRESPONDING ANGLES

$\angle ABC = \angle ACB$ [$\angle AXY = \angle AYX$]

AC = AB[SIDES OPP. TO EQUAL ∠S ARE EQUAL]

∴ΔABC is an isosceles triangle.

13. A model of an aeroplane is made to a scale of 1:400. Calculate:

(i) the length, in cm, of the model; if the length of the aeroplane is 40 m.

(ii) the length, in m, of the aeroplane, if length of its model is 16 m.

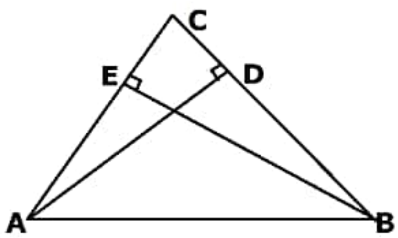
Ans. Scale = 1:400

(i) Length of model = length of the aeroplane/400 = 4000/400 = 10 cm

(ii) Length of the aeroplane = 400 Length of model= 400 0.16 = 4 16 = 64 m

14. The figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.

Show that:



I. $\Delta ADC \sim \Delta BEC$

II. CA CE = CB CD

III. $\Delta ABC \sim \Delta DEC$

IV. CD AB = CA DE

Ans. (i) In ΔADC & ΔBEC,

$\angle C = \angle C$ COMMON

$\angle ABE = \angle BEC = 90°$ GIVEN

$\Delta ADC \sim \Delta BEC$ [BY A.A.]

(ii) $\frac{CA}{CB} = \frac{CD}{CE}$ [CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION]

CA X CE = CB X CD

(iii) In ΔABC & ΔDEC,

$\angle C = \angle C$ COMMON

From (ii),

CA X CE = CB X CD

$\frac{CA}{CD} = \frac{CB}{CE}$

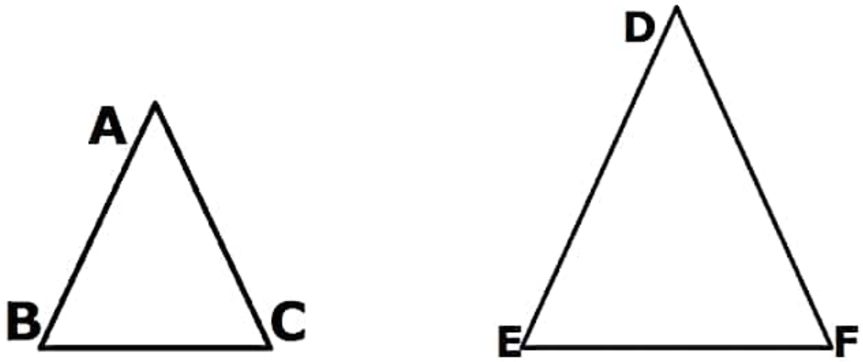
$\Delta ABC \sim \Delta DEC$[BY S.A.S.]

(iv) $\frac{CA}{CD} = \frac{AB}{DE}$ [CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION]

CD X AB =CA X DE

[PROVED]

15. In the given figure, Δ ABC is similar to Δ DEF, AB = (X – 0.5) cm, AC – 1.5X cm, DE = 9 cm, and DF = 3X cm. Find the lengths of AB and DF.



Ans. $\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{X-0.5}{9} = \frac{BC}{EF} = \frac{1.5X}{3X}$$

$$\frac{X-0.5}{9} = \frac{1}{2}$$

$2X-1 = 9$

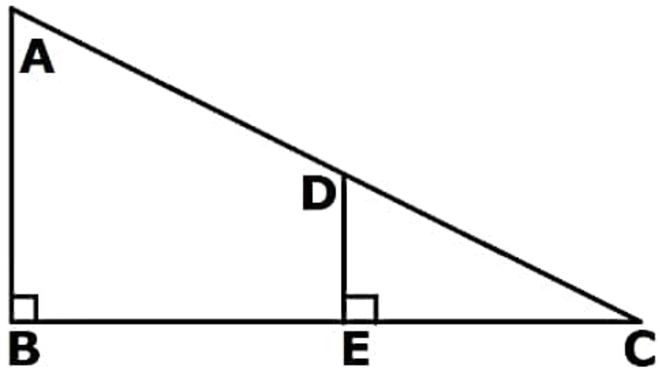
$2X = 10$

$X = 5$

$AB = X - 0.5 = 5 - 0.5 = 4.5$ cm

$DF = 3 \quad 5 = 15$ cm

16. From the figure, AB and DE perpendicular to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD.



Ans. In ΔABC & ΔDEC,

(i) $\angle ABC = \angle DEC$ [EACH 90°]

(ii) $\angle C = \angle C$ [COMMON]

$\therefore \Delta ABC \sim \Delta DEC$ [BY A.A.]

$\frac{AC}{DC} = \frac{AB}{DE}$ [CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION]

$\frac{24}{DC} = \frac{9}{3}$

DC = 8 cm

$\therefore AD = AC - DC = 24 - 8 = 16$ cm

17. In Δ ABC, D and E are points on the sides AB and AC respectively. Find whether DE // BC; if:

I. AD = 3 cm, BD = 4.5 cm, AE = 4 cm and AC = 10 cm

II. AB = 7 cm, BD = 4.5

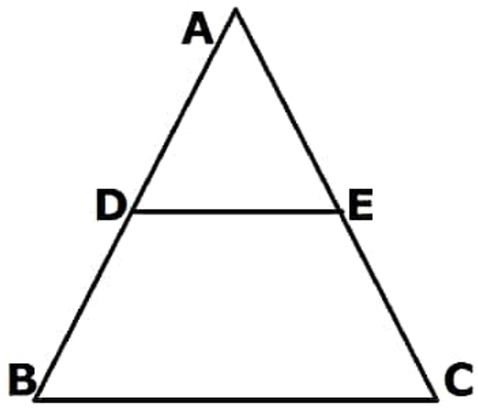
cm, AE = 3.5 cm and
CE = 5.6 cm

Ans. (i) CE = AC-AE = 10 -4 =6 cm   DE||BC.

$$\frac{AD}{BD}=\frac{3}{4.5}=\frac{2}{3}$$

$$\frac{AE}{CE}=\frac{4}{6}=\frac{2}{3}$$

$$\frac{AD}{BD}=\frac{AE}{CE}$$



(ii) $$\frac{AD}{BD}=\frac{AB-BD}{BD}=\frac{7-4.5}{4.5}=\frac{2.5}{4.5}=\frac{5}{9}$$

$$\frac{AE}{CE}=\frac{3.5}{5.6}=\frac{5}{8}$$

$$\frac{AD}{BD}\neq\frac{AE}{CE}$$

DE IS NOT PARALLEL TO BC.

18. A triangle ABC with AB = 3 cm, BC = 6cm and AC = 4cm is enlarged to ΔDEF such that the longest side of ΔDEF = 9 cm. Find the scale factor and hence, the lengths of the other sides of ΔDEF.

Ans. Let the longest side of ΔDEF be EF.

EF = 9 cm

Scale factor $=\frac{EF}{BC}=\frac{9}{6}=1.5$

DE = 3 X 1.5 = 4.5 cm

DF = 4 X 1.5 = 6 cm

19. The model of a building is constructed with the scale factor 1 : 30.

(i) If the model is 80 cm, find the actual height of the building in meters.

(ii) If the actual volume of a tank at the top of the building is 27 m^3, find the volume of the tank on the top of the model.

Ans. Scale factor = 1: 30

(i) Height of building = 30 X ht. of model

= 30 X 80 = 2400cm =24 m

(ii) Volume of tank on top of model =

$$\frac{\text{Volume of building}}{30 \times 30 \times 30} = \frac{27}{30 \times 30 \times 30} = 0.001 \text{ m}^3$$

Cylinder, Cone and Sphere

[CHAPTER – 20]

CYLINDER

1. The radius of a solid right circular cylinder decreases by 20% and its height increases by 10%. Find the percentage change in its: (i) volume (ii)curved surface area.

Ans. Let the radius of the cylinder= 100r

Height of the cylinder= 100h

Decrease in radius= 20/100 100r=20r

New radius= 100r-20r=80r

Increase in height= 10/100 100h= 10h

New height= 100h +10h= 110h

i)original volume= πr rh =1000000r rhπ

New volume= πr rh= 704000r rhπ

Decrease in volume= original vol.- new vol.= 296000r rhπ

% decrease in volume= Decrease in volume 100%/ original vol. =296000 /1000000 100 = 29.6%

ii) original csa= 20000rhπ

New csa= 176000rhπ

Decrease in csa= 2400rhπ

% decrease in csa= 2400rhπ/20000rhπ 100% = 12%

2. A circular tank of diameter 2 m is dug and the earth removed is spread uniformly all around the tank to form an embankment 2m in width and 1.6 m in height. Find the depth of the circular tank.

Ans. Let the depth of circular tank be x m.

Diameter of circular tank= 2m

- Its radius = d/2 = 2/2= 1m

Width of embankment= 2 m

Radius of embankment= 2+1= 3m

Height of embankment= 1.6 m

Volume of earth dug out= πr*rh = πx m^2

Volume of earth spread = Volume of earth dug out

$\pi(R^2 - r^2)h = \pi x$

$x = 8 * 1.6 = 12.8$ m

Depth of circular tank = 12.8 m

3. The height of a circular cylinder is 20 cm and the radius of the base is 7 cm. Find :

(i)volume (ii) total surface area

Ans. H = 20 cm

R = 7 cm

Volume of cylinder= $\pi r2h$ = 22/7 7 7 20 = 3080 cm3

TSA of cylinder = $2\pi r(h+r)$ = 2 22/7 7(20+7) =1188 cm2

4. A cylindrical tube, open at both ends, is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.4 cm in thickness. Calculate the volume of the metal correct to one place of decimal.

Ans. Internal radius = 11.2/2 = 5.6 cm

External radius = 5.6 cm +0.4 cm = 6.0 cm

Volume of metal = $\pi(R^2 - r^2)h = 22/7[(6^2 - (5.6)^2]* 21 = 306.2\ cm^3$

5. 50 identical circular plates are placed one above the other to form a solid cylinder. For each plate, diameter = 21 cm and thickness = 1.5 cm. Find :

(i) Curved surface area of cylinder formed

(ii) Volume of cylinder formed

Ans. Height of cylinder formed= 50   1.5 cm = 75 cm

Radius of cylinder = 21/2 = 10.5 cm

(i) CSA of cylinder = 2πrh = 2   22/7   10.5   75 = 4950 cm^2

(ii) Volume of cylinder = $\pi r^2 h$ = 22/7 $(10.5)^2$ 75= 25987.5 cm^3

6. Find the volume of the largest cylinder formed when a rectangular piece of paper 22 cm by 15 cm is rolled along its longer side.

Ans. Circumference of cylinder = 22 cm

Height of cylinder = 15 cm

Let the radius of cylinder = r cm

2πr = 22

2   22/7   r = 22

R = 7/2 cm

Volume of cylinder = $\pi r^2 h$= 22/7 $(7/2)^2$ 15 = 577.5 cm^3

7. A metal cube of side 11 cm is completely submerged in water contained in a cylindrical vessel with diameter 28 cm. Find the rise in the level of water.

Ans. Side of a metal cube = 11 cm

Volume of cube = 11^3 = 1331 cm^3

Radius of cylindrical vessel = 14 cm

Let the rise in water level be x cm.

Volume of cube submerged = Volume of water that rises by x cm

1331 = 22/7 14 14 x

On solving, x = 2.16 cm

Rise in water level = 2.16 cm

<u>CONE</u>

1. The capacity and the base area of a right circular conical vessel are 9856 cm^3 and 616 cm^2 respectively. Find the curved surface area of the vessel.
   Ans. Let radius of base and height of conical vessel be r cm and h cm respectively.

$\pi r^2 = 616$ and $\frac{1}{3}\pi r^2 h = 9856$

r = 14 cm and h = 48 cm

$l = \sqrt{48 \times 48 + 14 \times 14}$

$= \sqrt{2500}$

$= 50$ cm

$\therefore$ Curved surface area of vessel = $\pi rl = \frac{22}{7}$ 14 50 cm^2 = 2200 cm^2

2. Two right circular cones x and y are made. X having three times the radius of y and y having half the volume of x. Calculate the ratio between the heights of x and y.
Ans. Let radius of cone y be r.

Radius of cone x= 3r

Let volume of cone x be V.

Volume of cone y= V/2

For cone x:

$$H = \frac{V}{3\pi r^2}$$

For cone y:

$$H = \frac{V}{2\pi r^2}$$

∴Required ratio = 2:9

3. A solid cone of height 8 cm, and base radius 6 cm is melted and recasted into identical cones, each of height 2 cm and diameter 1 cm. Find the number of cones so formed.

Ans. Height of cone = 8 cm

R = 6 cm

Volume of cone = $96\pi\ cm^3$

Height of smaller cone = 2 cm

R = ½ cm

Volume of smaller cone = $\frac{\pi}{6} cm^3$

No. of cones formed= 576

4. The curved surface area of a cone is 12320 cm^2. If the radius of its base is 56 cm, find its height.

Ans. Radius of base = 56 cm

CSA of cone = 12320 cm^2

22/7 56 l = 12320

L= 70 cm

$H = \sqrt{70 \times 70 - 56 \times 56} = 42$ cm

Height = 42 cm

5. The diameters of two cones are equal. If their slant heights are in the ratio 5:4, find the ratio of their curved surface areas.

Ans. ATQ, Radius of first cone = Radius of 2^{nd} cone

Let their slant heights be 5x and 4x.

First cone:

Curved surface area = $\frac{55dx}{7}$

2^{nd} cone:

Curved surface area = $\frac{44dx}{7}$

Required ratio = 5:4

6. A vessel, in the form of an inverted cone, is filled with water to the brim. Its height is 32 cm and diameter of the base is 25.2 cm. Six equal solid cones are dropped in it, so That they are fully submerged. As a result, one- fourth of water in the original cone overflows. What is the volume of each of the solid cones submerged?

Ans.

Volume of vessel = Volume of water = 5322.24 cm^3

Volume of 6 solid cones= Volume of over flown water = 5322.24/4 = 1330.56 cm^3

∴Volume of each solid cone submerged = 1330.56/6 = 221.76 cm^3

7. Find what length of canvas, 2 m in width, is required to make a conical tent 12 m in diameter and 63 m in slant height?

   Also, find the cost of canvas at the rate of Rs.150 per metre.

Ans. Area of canvas required= πrl= 1188 m^2

Let the length of canvas required = x m.

L b= area

X 2= 1188

X = 594 m

Cost of canvas = length rate = 594 150= Rs. 89100.

SPHERE

1. The volume of a sphere is 38808 cm^3; find its diameter and the surface area.

Ans. Volume of sphere = 38808 cm^3

$$\frac{4}{3} \quad r^3 = 38808$$

$$r^3 = \frac{38808 \text{ X } 3 \text{ X } 7}{4 \text{ X } 22} = 441 \quad 21$$

r = 21 cm

∴ Diameter = 2r = 2 21 = 42 cm

∴ Surface area = 4 r^2 =4 $\frac{22}{7}$ 21 21 = 5544 cm^2

2. Eight metallic spheres; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.

Ans. Radius of each smaller sphere = 2 mm

Volume of each smaller sphere= $\frac{4}{3}$ r^3

$=\frac{4}{3} \qquad 2 \quad 2 \quad 2 = \frac{32\pi}{3}\ mm^2$

Volume of 8 smaller spheres = 8 $\quad \frac{32\pi}{3} = \frac{256\pi}{3}\ mm^2$

ATQ, Volume of the single sphere = $\frac{256\pi}{3}\ mm^2$

$$\frac{4}{3} \quad r^3 = \frac{256\pi}{3}$$

$r^3 = 64$

r = 4 mm

∴Radius of new sphere = 4 mm

3. If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetres in its volume, what is the diameter of the sphere?

Ans. Let the radius of the sphere be r.

$$\text{ATQ, } 4 \quad r^2 = \frac{4}{3} \quad r^3$$

$$\frac{r \times r \times r}{r \times r} = 3$$

$$r = 3$$

∴ Diameter = 2r =2 $\quad$ 3 = 6 cm

4. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted and recasted into a single solid sphere. Taking $\quad$ = 3.1, find the surface area of solid sphere formed.

Ans. Volume of the small metallic spheres = $\frac{4}{3}$ $[(6)^3+(8)^3$ $+(10)^3] = 2304$

Volume of the larger sphere = 2304

Let the radius of the larger sphere be R cm.

ATQ, $\frac{4}{3}$ $R^3 = 2304$

$$R^3 = \frac{2304*3}{4} = 1728$$

$$R = 12 \text{ cm}$$

Radius of solid sphere = 12 cm

∴Surface area of solid sphere formed = 4 r^2
= 4 3.1 12 12= 1785.6 cm^2

5. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Ans. Diameter of a sphere = 6 cm

Radius of the sphere = 3 cm

Volume of the sphere=$\frac{4}{3}$ r^3 = 36 cm^2

Diameter of wire = 0.2 cm

Radius of wire = 0.1 cm

Let length of wire= l.

Volume of wire = $r^2 l = \frac{\pi l}{100}$ cm^2

ATQ, 36 $= \frac{\pi l}{100}$

$l = 3600$ cm = 36 m

∴ Length of wire= 36 m

6. A solid sphere and a solid hemisphere have the total surface area. Find the ratio between their volumes.

Ans. Let the radius of sphere be R and that of hemisphere be r.

ATQ, $4 \quad R^2 = 3 \quad r^2$

$$r = \sqrt{\frac{4R*R}{3}}$$

∴ Ratio between their volumes = $\frac{4}{3} \quad R^3 / \frac{2}{3}(\sqrt{\frac{4R*R}{3}}) = 3\sqrt{3}$:

4

7. A certain number of metallic cones, each of radius 2 cm and height 3 cm, are melted and recast into a solid sphere of radius 6 cm. Find the number of cones used.

Ans. Radius of cone = 2 cm

Height of cone = 3 cm

Volume of each cone $= \frac{1}{3} \quad r^2 h = 4 \quad cm^3$

Radius of sphere = 6 cm

Volume of sphere = $\frac{4}{3} \quad r^3 = 288 \quad cm^3$

∴ No. of cones used $= \frac{\text{Volume of sphere}}{\text{Volume of each cone}} = \frac{288\pi}{4\pi} = 72$

Geometric Progression

[CHAPTER – 11]

Ex.1. Find the sum of first 20 terms of the G.P. 2, 4, 8, 16, ...

Sol. Given G.P. is 2, 4, 8, 16, ...

Here, $a = 2$ and $r = \dfrac{4}{2} = 2$

$$\left[\because r = 2, \text{ i.e. } r > 1\right]$$

Now, $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$\Rightarrow$ $S_{20} = \dfrac{2(2^{20} - 1)}{2 - 1}$

$= \dfrac{2(2^{20} - 1)}{1} = 2(2^{20} - 1)$

Ex.2. Find the sum of first n terms of the G.P. $1, \frac{2}{3}, \frac{4}{9}, \ldots$

Sol. Given G.P. is $1, \frac{2}{3}, \frac{4}{9}, \ldots$

Here, $a = 1,\ r = \dfrac{\frac{2}{3}}{1} = \dfrac{2}{3},\ r < 1$

$\therefore$ $S_n = \dfrac{a(1 - r^n)}{1 - r} = \dfrac{1\left[1 - \left(\frac{2}{3}\right)^n\right]}{1 - \frac{2}{3}} = \dfrac{1 - \left(\frac{2}{3}\right)^n}{\frac{3 - 2}{3}} = 3\left[1 - \left(\frac{2}{3}\right)^n\right]$

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Ex.3. How many terms of the G.P. 3, $\frac{3}{2}, \frac{3}{4}, \ldots$ are needed to give the sum $\frac{186}{32}$?

Sol. Here, $a = 3$ and $r = \dfrac{\frac{3}{2}}{3} = \dfrac{1}{2}$ $\quad [\because r < 1]$

Let $\quad S_n = \dfrac{a(1-r^n)}{1-r}$

$\Rightarrow \quad \dfrac{186}{32} = \dfrac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} \Rightarrow \dfrac{186}{32} = \dfrac{3\left(1-\frac{1}{2^n}\right)}{\frac{1}{2}} \Rightarrow \dfrac{186}{32} = 6\left(1-\dfrac{1}{2^n}\right)$

$\Rightarrow \quad \dfrac{31}{32} = 1-\dfrac{1}{2^n} \Rightarrow \dfrac{1}{2^n} = 1-\dfrac{31}{32} \Rightarrow \dfrac{1}{2^n} = \dfrac{1}{32}$

$\Rightarrow \quad 2^n = 2^5 \Rightarrow n = 5$

Note: To find first 3 terms in a G.P., we take them as $\frac{a}{r}$, a and ar.

Ex.4. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the common ratio and the first three terms.

Sol. Let $\frac{a}{r}$, a and ar be the first three terms of the G.P.

Then $\quad \dfrac{a}{r} + a + ar = \dfrac{13}{12}$ $\quad$ ...(i)

and $\quad \dfrac{a}{r} \times a \times ar = -1$

$\Rightarrow \quad a^3 = -1$

$\Rightarrow \quad a = -1$ $\quad$ [Considering only real roots]

From (i), we get $\quad -\dfrac{1}{r} - 1 - r = \dfrac{13}{12}$

$\Rightarrow \quad \dfrac{-1-r-r^2}{r} = \dfrac{13}{12}$

$\Rightarrow \quad -12-12r-12r^2 = 13r$

$\Rightarrow \quad 0 = 12r^2 + 25r + 12$

$\Rightarrow \quad 12r^2 + 25r + 12 = 0$

$\Rightarrow \quad 12r^2 + 16r + 9r + 12 = 0$

$\Rightarrow \quad 4r(3r+4) + 3(3r+4) = 0$

$\Rightarrow \quad (3r+4)(4r+3) = 0$

$\Rightarrow$ Either $\quad 3r + 4 = 0 \quad$ or $\quad 4r + 3 = 0$

$\Rightarrow \quad 3r = -4 \quad \Rightarrow \quad 4r = -3$

$\Rightarrow \quad r = -\dfrac{4}{3} \quad \Rightarrow \quad r = -\dfrac{3}{4}$

Thus, when $r = -\frac{3}{4}$, the three terms of the G.P. are $\frac{4}{3}, -1$ and $\frac{3}{4}$.

When $r = -\frac{4}{3}$, the three terms are $\frac{3}{4}, -1, \frac{4}{3}$.

Ex.5. Find the sum of the terms 7, 77, 777, 7777, ... to n terms.

Sol. This is not a G.P. However, we can relate it to a G.P. as follows:

$S_n = 7 + 77 + 777 + \ldots$ to n terms

$= 7(1 + 11 + 111 + \ldots$ to n terms$)$

Multiplying and dividing RHS by 9, we get

$= \dfrac{7}{9}(9 + 99 + 999 + \ldots$ to n terms$)$

$= \dfrac{7}{9}[(10-1) + (100-1) + (1000-1) + \ldots$ to n terms$]$

$= \dfrac{7}{9}[(10 + 100 + 1000 + \ldots n$ terms$) - (1 + 1 + 1 + \ldots n$ terms$)]$

$= \dfrac{7}{9}\left[\dfrac{10(10^n-1)}{10-1} - n\right]$ $\quad \left[\begin{array}{l}\because\ 10, 100, 1000, \ldots \text{ is a G.P. } a = 10, r = 10\ (r>1) \\ \therefore\ S_n = \dfrac{10(10^n-1)}{10-1}\end{array}\right]$

$= \dfrac{7}{9}\left[\dfrac{10(10^n-1)}{9} - n\right] = \dfrac{70}{81}(10^n-1) - \dfrac{7}{9}n$

Loci

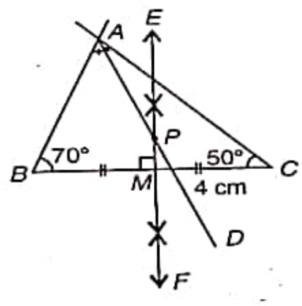
[CHAPTER – 16]

Question 1: Draw a ABC; having $\angle A = 60^0$ and $\angle B = 70^0$ and BC = 4cm . construct a point P, equidistant from AB and AC and also equidistant from points B and C.

Answer: $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow 60^0 + 70^0 + \angle C = 180^0$

$\therefore \angle C = 50^0$



(i) Draw ABC.

(ii) Draw bisector of $\angle A$ and $\perp$ bisector of the line BC. Both bisector meet at a point P.

Question 2: Ruler and compass only may be used in this question. ALL construction lines and arcs must be clearly shown and be sufficient length and clarity to permit assessment.

(i) Construct ABC, in which BC = 8 cm, AB =5 cm, $\angle ABC = 60^0$.

(ii) Construct the locus of points inside thetriangle which are equidistant from BA and BC

(iii) Construct the locus of points inside thetriangle which are equidistant from B and C .

(iv) Mark as P, the point which is equidistant from AB, BC and also equidistant

from B and C .

Answer:

(i) Draw ABC.

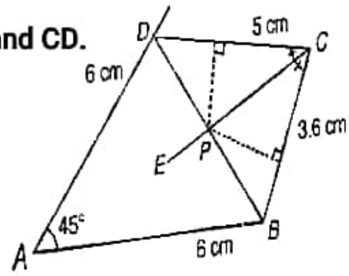
(ii) Draw bisector of BD of ∠ABC.
(iii) Draw⊥ bisector of line BC. (Say it EF)
(iv) Both bisectors meet at point P.
(v) Measure PB = 4.5 cm.

.

Question 3: Construct a quadrilateral ABCD in which ∠BAD = 45^0 , AD = AB = 6 cm, BC = 3.6 cm and CD = 5 cm.

(i) Measure ∠BCD.

(ii) Locate the point P on BD which equidistant from BC and CD.



Answer: Draw quadrilateral ABCD .

(i) Measure ∠BCD = 65^0 .

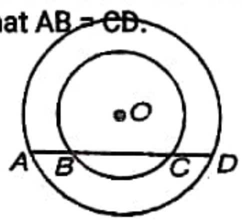
(ii) Drawbisector of ∠BCD, which intersects BD at P

Selection Test

Circles

[CHAPTER – 17]

Question 1: In the given figure, there are two concentric circles and AD is a chord of larger circle. Prove that AB = CD.



Answer : Draw OM⊥ AD. Now⊥ from the centre to the chord bisects the chord.

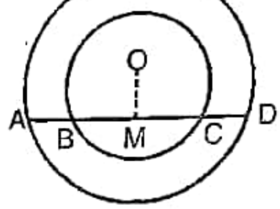
∴ AM = MD (for bigger circles) .....(i)

And BM = MC (for smaller circles).....(ii)

Subtracting (ii) from (i), we get

AM – BM =MD – MC

AB = CD



Question 2: Chords AB and *CD* of a circle with centre *O*, intersect at a point *E*. If *OE* bisects ∠AED, prove that *AB* = *CD*.

Answer : Draw OM ⊥ AB and ON ⊥ CD

$\angle 3 = \angle 4$ (each 90°)

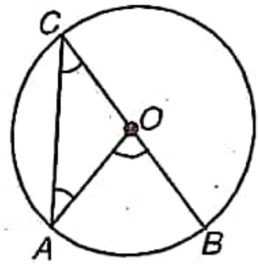
$\angle 1 = \angle 2$ (OE bisects $\angle AED$)

OE = OE (common)

$\Rightarrow \Delta OEM \cong \Delta OEN$ (*AAS* Congruence)

$\Rightarrow OM = ON$ (c.p.c.t.) $\Rightarrow AB = CD$

Question 3: In the given figure, O is the centre of the circle and $\angle AOB = 70°$, find the value of (i) $\angle OCA$ (ii) $\angle OAC$.



Answer :(i) $\angle AOB = 2\angle ACB$

$70° = 2\angle ACB \quad \Rightarrow \angle ACB = 35°$

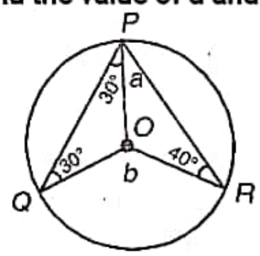
$\Rightarrow \angle OCA = 35°$

(*ii*) $OA = OC$ (radii)

$\Rightarrow \angle OCA = \angle OAC$

$\Rightarrow \angle OAC = 35°$

Question 4: In the given figure, O is the centre of the circle. Find the value of a and b



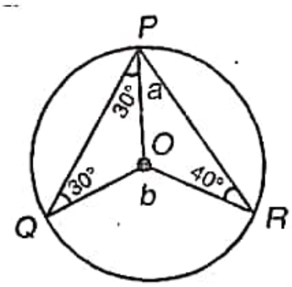
Answer : Join OP, OP = OQ (radii) $\therefore \angle OPQ = \angle OQP = 30°$

OP = OR (radii) $\therefore \angle OPR = \angle ORP = 40°$

$a = \angle OPQ + \angle OPR = 30° + 40° = 70°$

$b = 2a = 2 \times 70° = 140°$.

Question 5: In the given figure, POQ is a diameter of the O and $\angle POR = 110°$. Find $\angle QSR$.



Answer :$\angle POR + \angle QOR = 180°$ (Linear pair)

$\angle QOR = 70°$

Now $\angle QOR = 2\angle QSR$

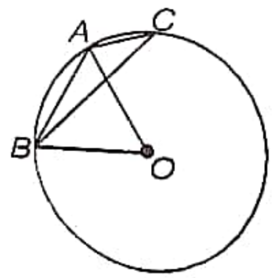
(Degree Measure Theorem)

$70° = 2\angle QSR$

$\Rightarrow \angle QSR = 35°$

Question 6: In the given figure, AB is a side of a regular hexagon and AC is a side of a regular octagon inscribed circle with centre O. Find :

(i) $\angle AOB$ (ii) $\angle ACB$ (iii) $\angle ABC$



Answer :(i) $\angle AOB = \dfrac{360°}{n} = \dfrac{360°}{6} = 60°$

(ii) $\angle AOB = 2\angle ACB$

$\Rightarrow \angle ACB = \dfrac{60°}{2} = 30°$

(iii) $\angle AOC = \dfrac{360°}{8} = 45°$

$\therefore \angle AOC = 2\angle ABC$

$\Rightarrow \angle ABC = \dfrac{45°}{2} = 22\frac{1}{2}°$

Question 7: AB is a diameter if the circle C(O, r) and OD $\perp$ AB. If C is any point on DB, find $\angle BAD$ and $\angle ACD$.

Answer :OD $\perp$ AB

∴ ∠AOD =∠BOD = 90°

∠BOD = 2 ∠BAD

(Degree Measure Theorem)

90° = 2∠BAD

⇒ ∠BAD = 45°

∠ADB= 90°(∠in a semicircle)

Now, ∠BAD +∠ABD + ∠ADB =180°

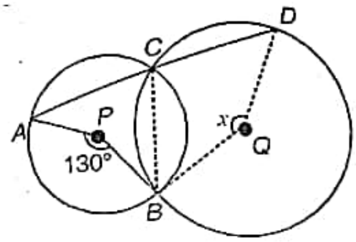
45°+∠ABD + 90°=180°

∠BAD = 45°

Also, ∠ACD =∠ABD[∠'s in same segment]

⇒ ∠BAD = 45°

Question 8: In the given figure, P and Q are centres of two circles intersecting at B and C. ACD is a straight line. Calculate the numerical value of x .



Answer :∠APB =2∠ACB ⇒ ∠ACB =65°

Now ∠ACB+ ∠BCD = 180°(Linear pair)

⇒ ∠BCD = 115° ∴ Reflex ∠BQD = 2∠BCD = 230°

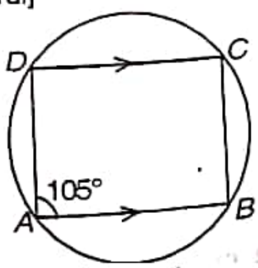
Also, x + Reflex ∠BQD = 360°

⇒ x +230° = 360°

⇒ x = 360° - 230° = 130°

Question 9: In the given figure, Find :

(i) ∠BCD (ii) ∠ADC (iii) ∠ABC



Answer :(i) ∠BCD + ∠BAD =180°[opposite ∠'s of a cyclic quadrilateral]

⇒ ∠BCD = 180° - 105° = 75°

(ii) ∠ADC + ∠BAD =180°[con. Int. ∠'s]

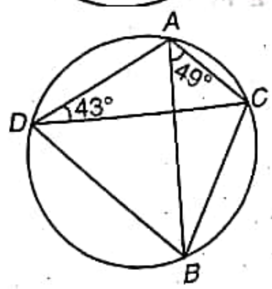
⇒ ∠ADC = 180° - ∠BAD = 180° - 105° = 75°

(iii)∠A + ∠B + ∠C + ∠D = 360°

⇒ ∠ABC = 360° - 75° - 75° - 105°

= 360° - 255° = 105°

Question 10: Find :(i) ∠CDB (ii) ∠ABC (iii) ∠ACB



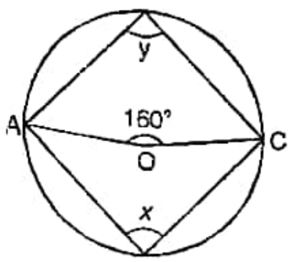
Answer :(i) ∠CDB = ∠BAC =49°

(ii) ∠ABC = ∠ADC =43°[∠'s in same segment]

(iii)Sum of ∠'s of a = 180°

49° + 43°+∠ACB =180° ⇒ ∠ACB = 88°

Question 11: In the figure, O is the centre of the circle and ∠AOC=160°. Prove that 3∠y-2∠x = 140°.



Answer :∠AOC =2x [Degree Measure Theorem]

160° =2x ⇒ x=80°

x + y=180°[Opp.∠'s of a cyclic quadrilateral]

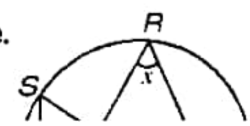
80°+y =180°

⇒ y=100°

L.H.S. =3y − 2x =3×100° − 2 × 80°=300°− 160°

=140° =R.H.S.

Question 12: For the given figure, find x if O is the centre of the circle.



Answer :$\angle QPS=90°$

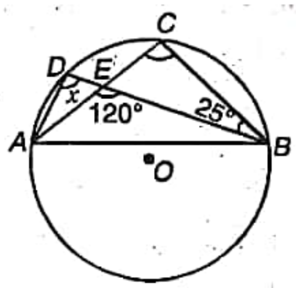
$\therefore\ \angle PSQ + \angle SQP + \angle QPS = 180°$

$\Rightarrow \angle PSQ = 55°$

Then $x=\angle PSQ$ ($\angle$'s in same segment)

$=55°$

Question 13: In the given figure, O is the centre of the circle, find the value of x .



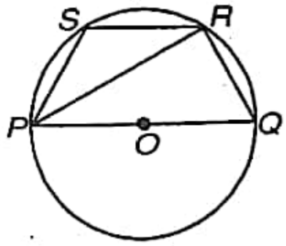
Answer :$x=\angle ACB$ ($\angle$'s in same segment)

$\angle AEB = \angle ACB + \angle CBE$

$\Rightarrow\ 120° = x +25°$

$\Rightarrow\ x =120° - 25° = 95°$

Question 14: PQRS is a cyclic quadrilateral in a circle with centre O. If $\angle PSR=130°$, Find $\angle QPR$.

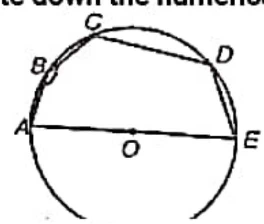


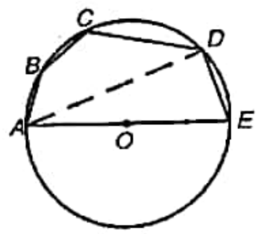
Answer :$\angle S + \angle Q = 180°$ and $\angle PRQ = 90°$ ($\angle$ in a semi circle)

$\Rightarrow\ Q =180° - 130° = 50°$

$\therefore\ \angle PSQ = 180° - (90° - 50°) = 40°$

Question 15: In the given figure, AOE is a diameter of circle, write down the numerical value of $\angle ABC + \angle CDE$, Give reasons of your answer.





Answer : Join AD $\angle ADE = 90°$ ($\angle$ in a semi circle)

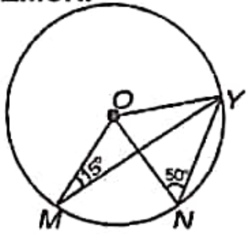
Also $\angle ABC + \angle ADC=180°$

[Opp.$\angle$'s of a cyclic quadrilateral]

Hence, $\angle ABC + \angle ADE + \angle ADC = 270°$

$\Rightarrow\ \angle ABC + \angle CDE = 270°$

Question 16: In the given figure, $\angle ONY = 50^0$ and $\angle OMY = 15^0$, find $\angle MON$.



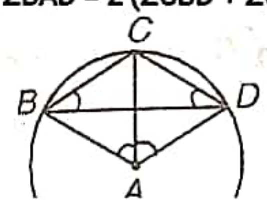
Answer :ON = OY (radii) $\therefore\ \angle ONY = \angle OYN = 50°$

Also OM = OY (radii) $\therefore\ \angle OMY = \angle OYM = 15°$

$\angle MYN = \angle OYN - \angle OYM = 35°$

$\angle MON = 2\angle MYN = 2\times 35° = 70°$

Question 17: In the quadrilateral ABCD, AB = AC = AD. Show that $\angle BAD = 2 (\angle CBD + \angle CDB)$.

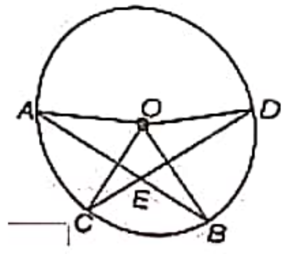


Answer : $\angle BAC = 2\angle CDB$

and $\angle DAC = 2\angle CBD$

Add (i) and (ii), we get ∠BAD = 2 [∠CBD + ∠CDB]

Question 18: In the given figure, O is the centre of the circle. Chord AB and CD intersect inside the circle at E. Prove that ∠AOC + ∠BOD = 2 ∠AEC .



Answer : Join CB, ∠AOC = 2∠ABC

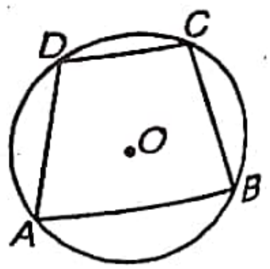
∠BOD = 2∠BCD

Add (i) and (ii)

∠AOC + ∠BOD = 2 (∠ABC + ∠BCD)

Proved ∠ABC + ∠BCD = 2 ∠AEC [Exterior ∠ of a]

Question 19: In the cyclic quadrilateral, if one pair of opposite sides is equal, the other pair is parallel.
Prove it .



Answer :AD = BC

∴ ∠AOD = ∠BOC

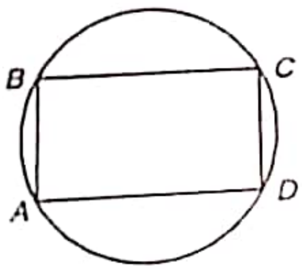
⇒ 2∠ACD = 2 ∠BAC

or ∠ACD = ∠BAC

But they are alternate interior ∠s

∴ AB || DC Proved.

Question 20: Prove that the parallelogram inscribed in a circle, is a rectangle.



Answer : In a ||gm opp. ∠ are equal and

also opp. ∠s of a cyclic quadrilateral are supplementary.

∴ ∠A = ∠ C

Also ∠A + ∠C = 180°

∠A + ∠C = 180°

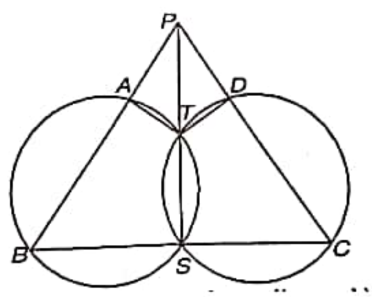
2∠A = 180°

⇒ ∠A = 90°

In ||gm ABCD,∠A = 90°

⇒ It is a rectangle.

Question 21: In the given figure, two circles intersect at S and T. STP, BSCand BAP are straight lines,
Prove that PATD is a cyclic quadrilateral.



Answer ∠PAT=∠PSB (Ext.∠ of a cyclic quad.) ....(i)

∠PDT = ∠PSC (Ext. ∠ of a cyclic quad.) ....(ii)

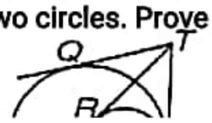
Adding (i) and (ii), we get

∠PAT + ∠PDT = 180° . Proved

Tangent to Circles

[CHAPTER – 18]

Question 1: Two circles touch internally at a point P and from a point T on the common tangent at P, tangent segments TQ and TR are drawn to the two circles. Prove that



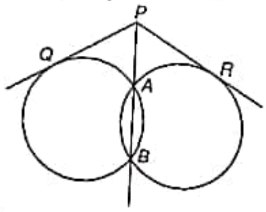
TQ = TR.

Answer: TP = TQ (Tangents are equal to a circle from an external point)

and TP = TR

$\Rightarrow$ TQ = TR

Question 2: In the given figure, two circles intersect each other at the points A and B. If PQ and PR are tangents to these circles from a point P on BA produced, Prove that PQ = PR.



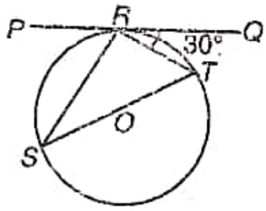
Answer: $PQ^2 = PA \times PB$ (PQ is a tangent and PBA is a secant)

and $PR^2 = PA \times PB$ (PR is a tangent and PBA is a secant)

$\Rightarrow$ $PQ^2 = PR^2$

$\Rightarrow$ PQ = PR

Question 3: If PQ is a tangent to a circle at R, calculate (i) ∠ROT (ii) ∠PRS , if ∠TRQ =30° and O is the centre of the circle.



Answer: (i) Join OR ∠RST = ∠TRQ (∠'s in alternate segment)

∠RST = 30°

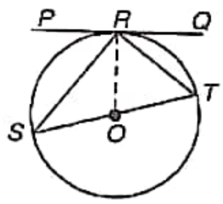
∠ROT = 2∠RST

= 60°

(ii) Now OR = OT (radii)

$\Rightarrow$ ∠ORT = ∠RTO

$\Rightarrow$ ∠RTO = 60°



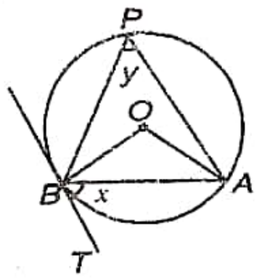
(∠ROT + ∠ORT + ∠RTO = 180°)

(60° + 2∠RTO = 180°)

∠PRS = ∠RTO (∠'s in alternate segment)

= 60°

Question 4: In the given figure, AB is a chord of a circle with centre O, BT is a tangent to the circle. If ∠OAB = 32°, find the values of x and y.



Answer:∠OAB = ∠OBA = 32°

OB = OA

∠OAB + ∠OBA + ∠AOB = 108°

∠AOB = 116°

∠AOB = 2y $\Rightarrow$ y = 58°

x = y (∠' s in alternate segment)

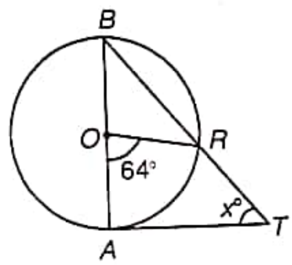
x = 58° . y = 58°

Question 5: In the given figure, AB is the diameter of the circle, with centre O and AT Is a tangent, find the value of x.

Answer:OA ⊥ AT $\Rightarrow$ ∠A = 90°(Radius is ⊥ to the tangent)

∠AOR = 2∠ABR

$\Rightarrow$ ∠ABR = 32°



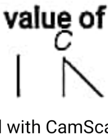
Now, in Δ ABT

∠BAT + ∠ABT + ∠ATB = 180°

90° + 32° + x° = 180°

$\Rightarrow$ x° = 180 − 122 =58° .

Question 6: ABC is a right-angled triangle, ∠B = 90°, with AB = 6 cm and BC = 8 cm. A Circle with center O has been inscribed inside the triangle. Find the value of x.



Answer: Using Pythagoras Theorem AC = 10 cm

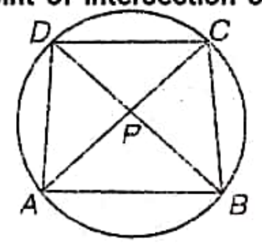
Join OA. OB and OC.

Then, ar (ΔAOB) + ar (ΔBOC) + ar (ΔAOC) = ar (ΔABC)

$\frac{1}{2}\times 6x+\frac{1}{2}\times 8x+\frac{1}{2}\times 10x= \frac{1}{2}(6\times 8) \Rightarrow x = 2$ cm.

(As OR, OP and OQ are respectively ⊥ to BC, AB, AB and AC. ∴ OR, OP and OQ are altitudes)

Question 7: In a trapezium ABCD, AB || CD and AD = BC: If P is a point of intersection of diagonals AC and BD, prove that PA x PC = PB × PD.



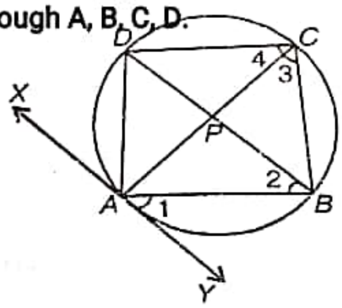
Answer: In trapezium ABCD, AD = BC

⇒ It is cyclic.

(If the non-parallel sides of a trapezium are equal, then it is a cyclic trapezium)

⇒ AC and BD are ------of a circle intersection each other at P∴ PA x PC = PB x PD.

Question 8: In a quadrilateral ABCD, the diagonal CA bisects the angle C. Prove that the diagonal BD is parallel to the tangent at A to the circle through A, B, C, D.



Answer: Also ∠4 =∠2 (∠'s in same segment)

∴ ∠4 =∠2

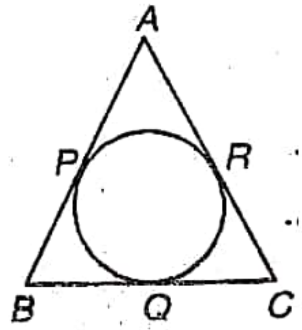
but ∠4 =∠2 (∠'s in alternate segment)

⇒ ∠4 =∠2

But they are alternate interior ∠'s ∴ XY || DB.

Question 9: The in circle of a Δ ABC touches BC, CA and AB at Q,R and P, respectively. Show that AP + BQ +CR = AR + BP +CQ = $\frac{1}{2}$ (Perimeter of ΔABC).

Answer: Tangents to a circle from an external point are of equal length.



∴AP = AR ......(i)

BQ =BP ......(ii)

And CR = CQ+AR ......(iii)

Adding (i), (ii) and (iii)

AP + BQ + CR = BP + CQ +AR ......(iv)

Now,

Perimeter (ΔABC) = AB+BC+AC

=AP+BP+BQ+QC+AR+CR

=AP + BQ + BQ + CR + AP + CR

Perimeter (ΔABC) = 2 (AP + BQ + CR)

Or $\frac{1}{2}$ Peri (ΔABC) = AP + BQ + CR .....(v)

(iv) And (v) ⇒ AP + BQ + CR = AR + BP + CQ

= $\frac{1}{2}$ Perimeter (ΔABC) Hence Proved.

Question 10: In the given figure. PA and PB are tangents to the circle. CE is a tangent to the

circle. At D. If AP = 15 cm. Find the perimeter of the triangle PEC.

Answer: PA = PB = 15 cm

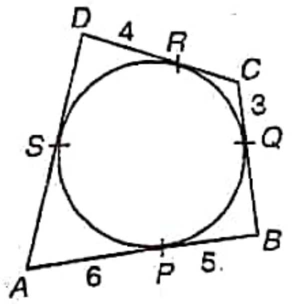
Perimeter of Δ PEC = PE + EC + PC

= PE + ED + DC + PC

= PE + EA + CB + PC (ED =EA and DC = CB)

= PA + PB = 15 + 15 = 30 cm.

Question 11: In the given figure, quadrilateral ABCD is circumscribed. Find the Perimeter of quadrilateral ABCD.



Answer: AP = AS = 6cm

BP = BQ = 5cm

CR = CQ =3cm

And DR = DS = 4 cm

Adding them. We get,

∴ APAP + BP + CR + DR = AS +BQ + CQ + DS

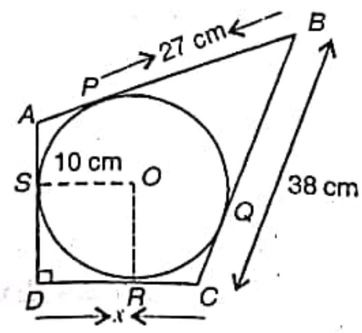
= 6 + 5 + 3 + 4 = 18 cm

∴ APAP + BP + CR + DR = AS + BQ + CQ + DS

= 18 + 18 =36cm

∴ APAB + BC + CD + DA = 36cm

Question 12: In the given figure, quadrilateral ABCD is circumscribed and AD ⊥ DC. Find x if radius of the incircle is 10 cm.



Answer: Join OR Radius is ⊥ to the tangent

⇒ ∠OSD = ∠ORD = 90°

OS = OR and each ∠ = 90°

⇒ OSDR is a square.

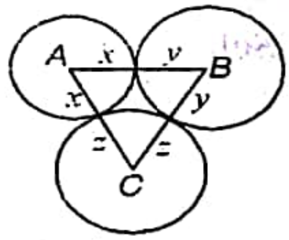
⇒ DR = 10 CM.

BQ = HP = 27 CM ⇒ CQ = 38 - 27 = 11 CM

CR = CQ =11 cm

X = DR + RC = 10+ 11 = 21cm

Question 13: Three circles with centres A, B and C touch each other externally. If AB= 5cm, BC = 7 cm and CA = 6 cm, find the radii of the three circl



Answer: Suppose radii of three circles be x, y and z cm.

AB + BC + AC = 18 cm

x + y + y + z + x = 18

x + y + z = 9 ....(i)

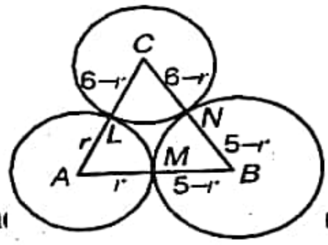
Now x+y=5 ....(ii)

And y + z = 7 cm ....(iii)

From (i) and (ii), we get z = 4 cm

From (iii) and (ii), we get x=2 cm

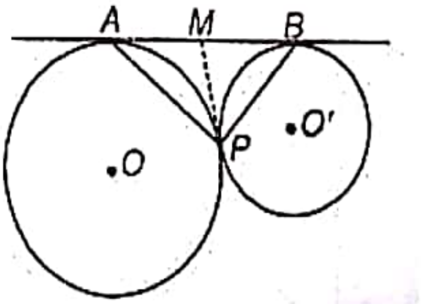
From (ii), we get y = 3 cm



Question 14: In the given figure, two figure, two circles touch ea oint

P. AB is the direct common tangent of these circle. Prove that:
(i) $\angle APB = 90°$ (ii) tangent at point P bisects AB

Answer: (i) AM = MP (Tangents from an external point)



$\Rightarrow \angle MAP = \angle MPA$

Similarly, MP = MB

$\therefore \angle MBP = \angle MPB$

Now in ΔABP.

$\angle PAB + \angle ABP + \angle APB = 180°$

$\angle MPA + \angle MPB + \angle MPA +$.

$2(\angle MPA + \angle MPB) = 180°$

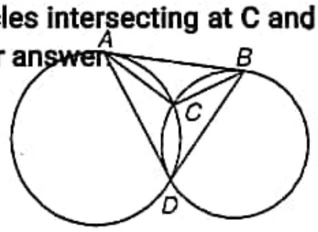
$\Rightarrow \angle APB = 90°$

(ii) AM = MP and MP = MB

$\Rightarrow$ AM = MB

$\Rightarrow$ Tangent at P bisects AB.

Question 15: In the given figure, AB is a common tangent to two circles intersecting at C and D. Write down the measure of $\angle ACB + \angle ADB$. Justify your answer.



Answer: $\angle 1 = \angle 4$ ($\angle$'s in alternate segment)

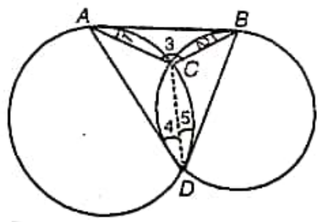
And $\angle 2 = \angle 5$ ($\angle$'s in alternate segment)

$\angle 1 + \angle 2 + \angle 3 = 180°$ (sum of angles of a triangle)

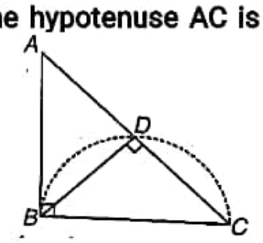
$(\angle 4 + \angle 5) + \angle 3 = 180°$

$\Longrightarrow \angle ADB + \angle ACB = 180°$

or $\angle ACB + \angle ADB = 180°$



Question 16: In a right-angled triangle ABC, the perpendicular BD on the hypotenuse AC is drawn. Prove that
(i) $AC \times AD = AB^2$ and (ii) $AC \times CD = BC^2$.



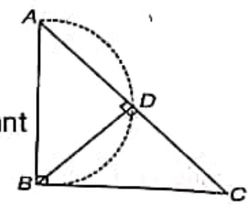
Answer: Taking AB and BC as diameter, draw semi-circles.

(i) In the given figure, AB is a tangent and ADCis a secant.

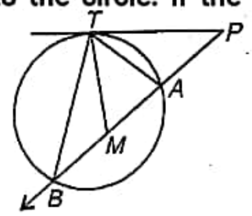
$\therefore AB^2 = AD \times AC$

(ii) Similarly, in the given figure, CB is a tangent and CDA is a secant

$\therefore BC^2 = CD \times CA$



Question 17: In the given figure, PT is a tangent and PAB is a secant to the circle. If the bisector of $\angle ATB$ intersects AB at M. Prove that
(i) ($\angle PMT = \angle PTM$ (ii) PT = PM.



Answer: $\angle 1 = \angle 4$ ($\angle$'s in alternate segment) ....(i)

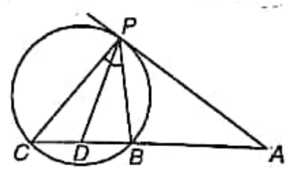
$\angle 2 = \angle 3$ (TM bisects $\angle ATB$) ....(ii)

(i) + (ii) $\Rightarrow$ $\angle 1 + \angle 2 = \angle 3 + \angle 4$

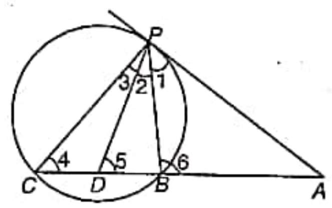
$\angle 1 + \angle 2 = \angle 5$

Or$\angle PTM = \angle 5$ (Exterior $\angle$ of a Δ)
Or$\angle PTM = \angle PMT$
$\Rightarrow$ PM = PT (sides opp. to equal angles)

Question 18: In the given figure, AP is a tangent to the circle at P. ABC is a secant such that PD is bisector of $\angle BPC$.Prove that: $\angle BPD = \frac{1}{2}[\angle ABP - \angle APB]$.

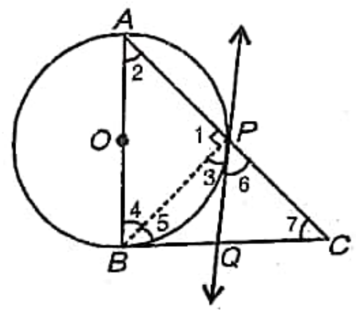


Answer:$\angle 1 = \angle 4$ ($\angle$'s in alternate segment)
$\angle 5 = \angle 3 + \angle 4$ (Exterior $\angle$ of a Δ)
And $\angle 2 = \angle 3$(PD bisicts $\angle BPC$)
$\angle 6 = \angle 2 + \angle 5$(Exterior $\angle$ of a $\triangle$)
Or $\angle 6 = \angle 2 + \angle 3 + \angle 4$
$\angle 6 = \angle 2 + \angle 2 + \angle 1$
Or $\angle 6 - \angle 1 = 2\angle 2 \Rightarrow \angle 2 = \frac{1}{2}(\angle 6 - \angle 1)$
Or $\angle BPD = \frac{1}{2}(\angle ABP - \angle APB)$

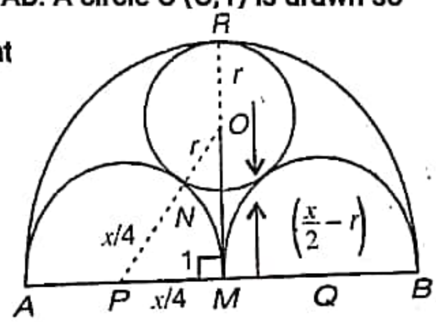


Question 19: In right angled $\triangle ABC$, a circle with side AB as diameter is drawn to intersect the hypotenuse AC at P. Prove that tangent to the circle at P bisects the side BC.

Answer:$\angle 3 = \angle 2$($\angle$' s in alternate segment)
$\angle 1 = 90°$ ($\angle$ in a semi-circle)
$\Rightarrow \angle 3 + \angle 6 = 90°$ ...(i)
also, $\angle B = 90°$
$\therefore$ In ΔABC.
$\angle 2 + \angle 7 = 90°$ ...(ii)
As $\angle B = 90°$
$\angle 2 + \angle B + \angle 7 = 180°$



Question 19: AB is a line segment and M is its mid-point. Semi-circles are drawn with AM, MB and AB as diameters on the same side of the line AB. A circle C (O, r) is drawn so that it touches all the three semi-circles. Prove that

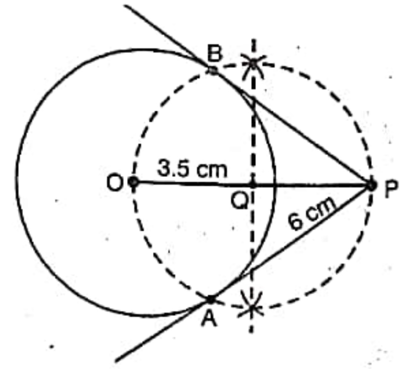


Answer:$\angle 1 = 90°$ (Radius is $\perp$ to the tangent)
Let AB = x, then AM = $\frac{x}{2}$ and PM = PN = $\frac{x}{4}$
Using Pythagoras Theorem
$OP^2 = PM^2 + OM^2$
$$\left(\frac{x}{4}+r\right)^2 = \left(\frac{x}{4}\right)^2 + \left(\frac{x}{2}-r\right)^2$$
$$\frac{x^2}{16} + r^2 + \frac{xp}{2} = \frac{x^2}{16} + \frac{x^2}{4} + r^2 - xr$$
$R = \frac{1}{6}x$ or $r = \frac{1}{6}AB$.

Construction of Circles
[CHAPTER – 19]

Question 1:Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the center. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.

Answer: Steps of construction:



(i) Draw a circle of radius 3 cm with centre O.

(ii) Take a point P outside the circle such that OP = 6 cm.

(iii) Draw ⊥ bisector of OP which intersect OP at Q.

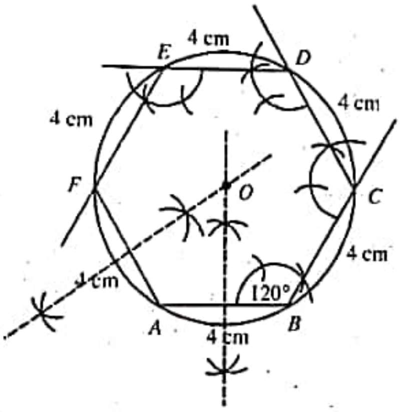
(iv) Draw a circle of radius OQ and centre Q

which cut the given circle at A and B.

(v) Join P to A and B to get the required tangents PA and PB.

(vi) Measure the length PA = PB = 4.5 cm

Question 2: Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.



Answer: Steps of construction:

(i) Draw a line segment *AB* = 4 cm, and draw a Regular hexagon by taking interior angle of 120°

(ii) Draw perpendicular bisector of *AB* and *AF* which meet at a point *O*, take *O* as a centre, and *OA* a radius draw circle.

Question 3:Use ruler and compasses only for this question

(i) Construct ΔABC, where AB = 3.5 cm, BC =6 cm and ∠ABC = 60

(ii) Construct the locus of points inside the trianglewhich are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

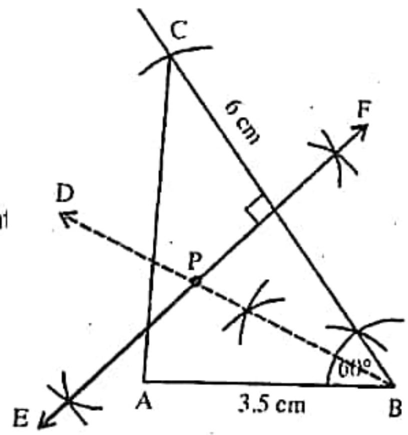
(iv) Mark the point P which is equidistant from AB.BC and also equidistant from B and C. Measureand record the length of PB.

Answer:(i) ΔABC is the required triangle in which AB = 3. cm, ∠B = 60° , BC 6 cm.

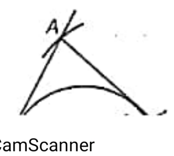
(ii) BD, the angle bisector of ∠B is the locus of points Inside the triangle which are equidistant from BA and BC.

(ii) EF, the perpendicular bisector of BC is the locus ofpoint inside the triangle which are equidistant fromB and C.

(iv) PB = 3.5 cm.



Question 4: Using a ruler, construct a triangle ABC with BC = 6.4 cm. CA = 5.8 cm and ∠ABC = 60°. Draw its incircle. Measure and record the radius of the incircle.



Answer:

(i) Draw BC=6.4 cm.

(ii) Make ∠ABC = 60° at point B.

(iii) Cut AC= 5.8 cm, Join C to A make a ABC.

(iv) Draw angle bisectors of ∠B and ∠C which intersects at O.

(v) Draw OD ⊥BC .

(vi) Take O as a centre and OD as a radius, draw the circle.

Question 5: Construct a circle in a regular hexagon of side 3 cm.

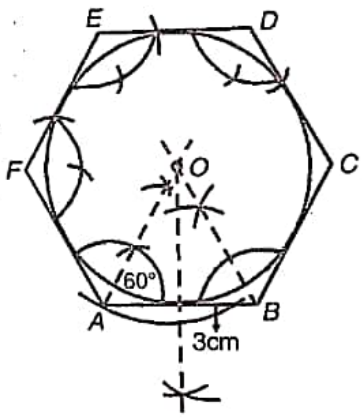
Answer:

(i) Draw the hexagon of side 3 cm.

(ii) Draw bisectors of any two angles, let these meets at O.

(iii) From O, draw ⊥ on any side say it ON.

(iv) Take O as a centre and ON as a radius, draw the circle.



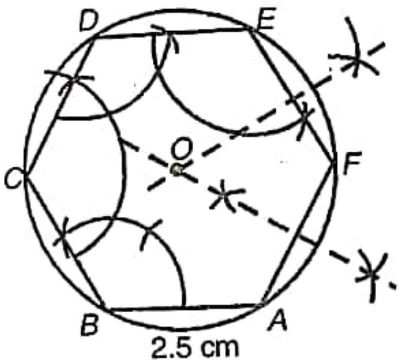
Question 6: Construct a circle about a regular hexagon of side 2.5 cm.

Answer:

(i) Construct a hexagon of side 2.5 cm.

(ii) Draw the ⊥ bisector of any two sides,
let these meet at a point *O*.

(iii) With *O* as a centre and radius equal to the
distance of *O* from any vertex draw a circle.



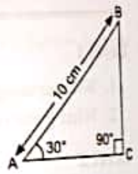
Heights and Distance
[CHAPTER – 22]

Ex.1. In Δ ABC, $\angle C = 90°$, $\angle A = 30°$ and AB = 10 cm. Find the remaining angle and sides.

Sol. We know that $\angle A + \angle B + \angle C = 180°$ [$\because$ Sum of angles of triangle is 180°]

$\Rightarrow$ $\angle B = 60°$

In right-angled ΔACB, $\frac{BC}{AB} = \sin 30° \Rightarrow BC = 10 \times \frac{1}{2} = 5$ cm

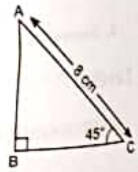


and $\frac{AC}{AB} = \cos 30°$

$\Rightarrow$ $AC = \frac{10 \times \sqrt{3}}{2} = 5\sqrt{3}$ cm

Ex.2. Find the area of Δ ABC in which $\angle ABC = 90°$, $\angle ACB = 45°$ and AC = 8 cm.

Sol. In right-angled ΔABC, $\frac{AB}{AC} = \sin 45° \Rightarrow AB = 8 \times \frac{1}{\sqrt{2}} = 4\sqrt{2}$ cm

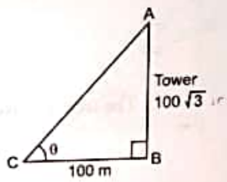


and $\frac{BC}{AC} = \cos 45° \Rightarrow BC = 8 \times \frac{1}{\sqrt{2}} = 4\sqrt{2}$ cm

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2} = 16$ cm^2

Ex.3. A tower is $100\sqrt{3}$ m high. Find the angle of elevation of its top from a point 100 m away from its foot.

Sol. In right-angled ΔABC, $\frac{AB}{BC} = \tan \theta$



$\Rightarrow$ $\frac{100\sqrt{3}}{100} = \tan \theta$

$\Rightarrow$ $\sqrt{3} = \tan \theta$

$\Rightarrow$ $\tan 60° = \tan \theta$

$\Rightarrow$ $\theta = 60°$

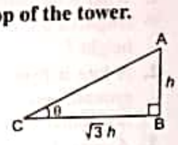
$\therefore$ Angle of elevation of its top from a point 100 m away from its foot = 60°

Ex.4. The shadow of a tower is $\sqrt{3}$ times its height. Find the angle of elevation of the top of the tower.

Sol. Let height of the tower be h units

Then length of the shadow = $\sqrt{3}h$ units

In right-angled ΔABC, $\frac{AB}{BC} = \tan \theta$



$\Rightarrow$ $\frac{h}{\sqrt{3}h} = \tan \theta$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \tan \theta$

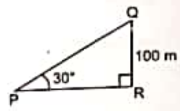
$\Rightarrow$ $\tan 30° = \tan \theta$

$\Rightarrow$ $\theta = 30°$

$\therefore$ Angle of elevation of top of tower = 30°

Ex.5. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, how far is P from the foot of the tower?

Sol. In right-angled ΔQRP, $\frac{PR}{QR} = \cot 30°$



$\Rightarrow$ $PR = 100\sqrt{3}$

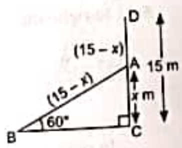
$\Rightarrow$ $PR = 173.2$ m

$\therefore$ P is 173.2 m far from the foot of tower.

Ex.6. A vertical straight tree 15 m high is broken by wind in such a way that its top touches the ground and makes an angle of 60° with the ground. At what height from the ground, did the tree break ?

Sol. Suppose the tree is broken at a height x m above the ground.

In right-angled ΔACB, $\frac{AC}{AB} = \sin 60°$



$\Rightarrow$ $\frac{x}{15 - x} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $2x = 15\sqrt{3} - \sqrt{3}x$

$\Rightarrow$ $2x + \sqrt{3}x = 15\sqrt{3}$

$\Rightarrow$ $x(2 + \sqrt{3}) = 15\sqrt{3}$

$\Rightarrow$ $x = \frac{15\sqrt{3}}{2 + \sqrt{3}}$

$\Rightarrow$ $x = \frac{15\sqrt{3}}{(2 + \sqrt{3})} \times \frac{(2 - \sqrt{3})}{(2 - \sqrt{3})}$ [Rationalising the denominator]

$\Rightarrow$ $x = \frac{15\sqrt{3}(2 - \sqrt{3})}{4 - 3}$

$\Rightarrow \quad x = 30\sqrt{3} - 45$

$\Rightarrow \quad x = 30 \times 1.73 - 45$

$\Rightarrow \quad x = 3 \times 17.3 - 45$

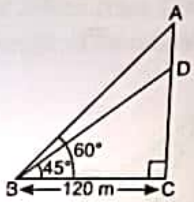
$\Rightarrow \quad x = 51.9 - 45$

$\Rightarrow \quad x = 6.9$ m

$\therefore$ The tree is broken at the height of 6.9 m from the ground.

Ex.7. The angle of elevation of the top of an unfinished tower at a point 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60°?

Sol. In right-angled ΔDCB, $\quad \dfrac{DC}{BC} = \tan 45° \quad \Rightarrow DC = BC = 120 \text{ m}$



In right-angled ΔACB, $\quad \dfrac{AC}{BC} = \tan 60°$

$\Rightarrow \quad AC = 120\sqrt{3} = 207.84 \text{ m}$

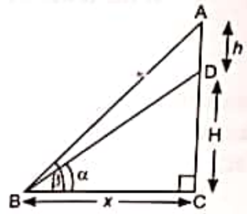
$\therefore$ Height at which the tower must be raised $= AD = AC - DC$

$= 207.84 \text{ m} - 120 \text{ m}$

$= 87.84 \text{ m}$

Ex.8. **A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 'h'. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Prove that the height of the tower is $\dfrac{h \tan\alpha}{\tan\beta - \tan\alpha}$.**

Sol. Let $BC = x$ units and $DC = H$ units



In right-angled ΔDCB, $\quad \dfrac{DC}{BC} = \dfrac{H}{x} = \tan\alpha \quad \Rightarrow x = \dfrac{H}{\tan\alpha}$ ...(*i*)

In right-angled ΔACB, $\quad \dfrac{AC}{BC} = \dfrac{h + H}{x} = \tan\beta \quad \Rightarrow h + H = x \tan\beta$

$\Rightarrow \quad h + H = \dfrac{H}{\tan\alpha} \tan\beta$ [From (*i*)]

$\Rightarrow \quad h \tan\alpha + H \tan\alpha = H \tan\beta$

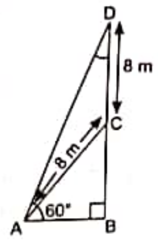
$\Rightarrow \quad H \tan\beta - H \tan\alpha = h \tan\alpha$

$\Rightarrow \quad H = \dfrac{h \tan\alpha}{\tan\beta - \tan\alpha}$

$\therefore$ Height of the tower is $\dfrac{h \tan\alpha}{\tan\beta - \tan\alpha}$ Hence proved.

Ex.9. **A ladder 8 m long reaches a point 8 m below the top of a vertical flagstaff. From the foot of the ladder, the angle of elevation of the top of the flagstaff is 60°. Find the height of the flagstaff.**

Sol. In right-angled ΔABD, $\quad \angle A + \angle B + \angle D = 180°$ [Sum of angles of triangle is 180°]



$\Rightarrow \quad \angle D = 30°$

Now, in right-angled ΔACD, $\quad CD = CA$

$\Rightarrow \quad \angle CAD = \angle ADC = 30°$ [$\because$ Angles opposite to equal sides of triangle are equal]

Also, $\quad \angle BAC = \angle BAD - \angle CAD$

$= 60° - 30° = 30°$

In right-angled ΔABC, $\quad \dfrac{BC}{AC} = \sin 30°$

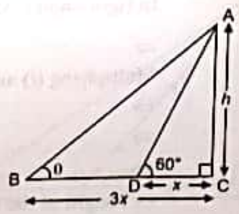
$\Rightarrow \quad BC = 8 \times \dfrac{1}{2} = 4 \text{ m}$

$\therefore$ Height of the flagstaff $= DB = DC + BC = 8 + 4 = 12 \text{ m}$

Ex.10. **The shadow of a flagstaff is three times as long as the shadow of a flagstaff, when the sun rays meet the ground at an angle of 60°. Find the angle between the sun rays and the ground at the time of longer shadow.**

Sol. Suppose the angle is θ.

Let $DC = x$ units, then $BC = 3x$ units and $AC = h$ units



In right-angled ΔACD, $\quad \dfrac{AC}{DC} = \dfrac{h}{x} = \tan 60° \quad \Rightarrow \quad h = \sqrt{3}x$ ...(*i*)

In right-angled ΔACB, $\quad \dfrac{AC}{BC} = \tan\theta$

$$\Rightarrow \quad \frac{h}{3x} = \tan\theta$$

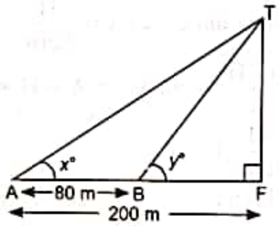
$$\Rightarrow \quad \frac{\sqrt{3}x}{3x} = \tan\theta \qquad \text{[From (i)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \tan\theta$$

$$\Rightarrow \quad \tan 30° = \tan\theta \Rightarrow \theta = 30°$$

$\therefore$ Angle between the sun rays and the ground at the time of longer shadow = 30°.

Ex. 11. In the given figure (not drawn to scale), TF is a tower. The elevation of T from A is $x°$, where $\tan x° = \frac{2}{5}$ and AF = 200 m. The elevation of T from B is $y°$, where AB = 80 m. Calculate (*i*) the height of the tower TF, (*ii*) the angle '*y*' correct to nearest degree.



Sol. (*i*) In right-angled ΔTFA, $\frac{TF}{AF} = \tan x° \Rightarrow \frac{TF}{200} = \frac{2}{5} \Rightarrow TF = 80$ m

$\therefore$ Height of tower TF = 80 m

(*ii*) We have BF = AF – AB = 200 – 80 = 120 m

In right-angled ΔTFB, $\frac{TF}{BF} = \tan y°$

$$\Rightarrow \quad \frac{80}{120} = \tan y°$$

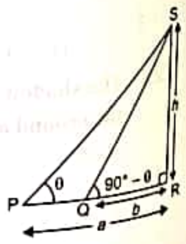
$$\Rightarrow \quad \tan y° = \frac{2}{3} = 0.6667$$

$$\Rightarrow \quad \tan y° = \tan 34°$$

$$\Rightarrow \quad y° = 34°$$

Ex. 12. The angles of elevation of the top of a tower from two points P and Q at distance of '*a*' and '*b*' respectively from the base and in the same straight line with it are complementary. Prove that height of the tower is $\sqrt{ab}$.

Sol. Let $\angle SPR = \theta$, then $\angle SQR = 90° - \theta$ and SR = h units



In right-angled ΔSRP, $\frac{SR}{PR} = \frac{h}{a} = \tan\theta$

$$\Rightarrow \quad h = a\tan\theta \qquad ...(i)$$

In right-angled ΔSRQ, $\frac{SR}{QR} = \frac{h}{b} = \tan(90° - \theta)$

$$\Rightarrow \quad h = b\cot\theta \qquad ...(ii)$$

Multiplying (*i*) and (*ii*), we get

$$\Rightarrow \quad h^2 = ab\tan\theta\cot\theta$$

$$\Rightarrow \quad h^2 = ab \qquad [\because \tan\theta \cdot \cot\theta = 1]$$

$$\Rightarrow \quad h = \sqrt{ab}$$

$\therefore$ Height of the tower is $\sqrt{ab}$.

Hence proved.

SELECTION test over